

PUPILS' PROCESSES OF THINKING: LEARNING TO SOLVE ALGEBRAIC PROBLEMS IN ENGLAND AND THAILAND

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This paper presents some key findings concerning pupils' processes of thinking in simplifying algebraic problems. The findings are drawn from an algebra test given to 103 English pupils and 186 Thai pupils. This paper focuses on one of the six themes explored during the study. The simplification theme has been split into level one: simplify one variable, level two: simplify two variables, level three: simplify two variables with brackets, and level four: simplify two variables of second order with brackets. The pupils' processes in simplification were categorised from their written responses. English and Thai pupils used the same processes to simplify the level one question. The differences were increased when they faced the harder items. The main difficulties were dealing with negative signs and multiplying out the brackets.

OVERVIEW

Basic education in England is organised on the basis of four Key Stages: Key Stage 1 caters for pupils aged 5-7 (Year 1-2), Key Stage 2 for pupils aged 7-11 (Year 3-6), Key Stage 3 for pupils aged 11-14 (Year 7-9), and Key Stage 4 for pupils aged 14-16 (Year 10-11). In Thailand, it is comprised 6 years for primary education (Primary 1-6, aged 6-12), 3 years for lower secondary education (Secondary 1-3, aged 12-15), 3 years for upper secondary education (Secondary 4-6, aged 15-18).

The design of the test was based on the algebra contents in the National Numeracy Strategy: Framework for teaching mathematics year 7, 8, and 9 in England and on the mathematics curriculum for the lower secondary level in Thailand. The test seeks to illustrate pupils' processes of thinking for each item at different levels of difficulties. There are 6 themes altogether, patterns/sequences, simplification, substitution, equations, functions/graphs, and word problems. Within each theme, there are 4 levels of difficulty ranging from the easiest level to the most difficult one.

The second theme of the test, simplification, is the process of adding and subtracting like terms in an expression. Like terms are those having exactly the same letters and exponents. They may differ only in their coefficients. This theme was tested using four questions, designed to observe the pupils' processes of thinking as they manipulated the like terms in different forms of expression. The questions are shown below:

Simplification

- Item 2** Simplify the expression $2a - a + 3a$. (*Level one: Simplify one variable*)
- Item 8** Simplify the expression $6 + 3b - c - 6b - c + 2$. (*Level two: Simplify two variables*)
- Item 14** Simplify $3p + 5(p-3) - 2(q-4)$. (*Level three: Simplify two variables with brackets*)
- Item 20** Multiply out the bracket and then simplify $x^2 + 2xy - 3(xy - 2x^2)$. (*Level four: Simplify two variables with second order and brackets*)

Level one: Simplify a one variable expression. Item2 was designed to observe how pupils simplify a one variable expression. *Level two:* Simplify a two variable expression. Item8 was to investigate how they manipulate those two variables. *Level three:* Simplify a two variable with brackets expression. Item14 was to examine pupils' processes when they faced the expression with brackets. *Level four:* Simplify a two variable expression with second order and brackets included. Item20 was to gain insight into their views of variables in different forms.

The pupils' processes of thinking in handling simplification problems were categorised from their responses as correct conception processes, misconception processes, and incomplete response processes.

Correct conception processes are the methods that show the way to simplify like terms in the expression, and multiply out the brackets whether they obtained the correct answer or not.

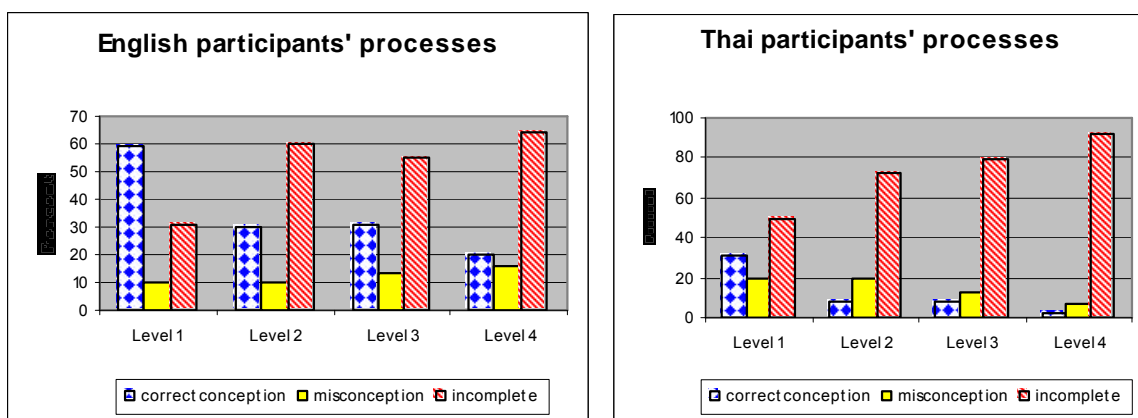
Misconception processes are those in which pupils attempt to simplify unlike terms, omit brackets, multiply only the first term in the brackets, and attempt to set up an equation or carry out substitutions. In these processes, they obtained the incorrect answers.

Incomplete response processes are those that show the attempt to work it out but did not reach completion, those that gave the answer without working, and those that made no response to the question or did not reach that question. Some correct answers appeared without working.

A COMPARISON OF PUPILS' PROCESSES OF THINKING IN SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS BETWEEN ENGLAND AND THAILAND

The next figures show the processes that the English and Thai pupils used in approaching these problems in each level of difficulty.

Figure1: Percentages by process at each level of difficulty.



The figures are presented as the percentages of pupils' processes of thinking in approaching the problems at each level of difficulty.

As shown in the bar charts above, 59.2 % of English pupils and 31.2 % of Thai pupils used the correct conception processes to solve the *level one* problem. There was a large drop between *level one* and *level two* of those making up the correct conception group in both countries. 30.1 % of English participants and 7.5 % of Thai participants used correct conception process. There was a minimal increase to 31.0 % among English pupils and an increase to 8.1 % among Thai pupils using the correct conception methods to solve the *level three* question. In the *level four* question, Thai pupils used correct conception process in only 2.2 % of cases while for English pupils the figure was 20.4 %.

The results indicate that Thai pupils had more misconception and incomplete responses than English pupils. These related to their taught experience. More than 50 % of the participants in both countries made incomplete responses to the *level two*, *level three* and *level four* questions.

The following sections describe the sub-processes, which pupils used in *level one* problem.

Level one Item 2 Simplify the expression $2a-a+3a$.

The *level one* question was designed to examine pupils' processes when manipulating a one variable expression. Pupils' responses were categorised into three groups as correct conception, misconception, and incomplete response.

Table1: Percentages of processes used and outcomes for simplification level one.

Processes Simplification Level one	England		Thailand	
	Used correct	Used correct	Used correct	Used correct
Correct conception	59.2	96.7	31.2	84.5
Incorrect operation	1.0	0.0	4.3	0.0
Letter ignored	3.9	100.0	0.0	0.0
Left to right	48.5	100.0	25.3	100.0
Plus to minus	5.8	83.3	1.6	66.7
Misconception	9.7	0.0	19.4	0.0
Incorrect operation	1.0	0.0	5.9	0.0
Letter ignored	6.8	0.0	5.4	0.0
Substitution	1.9	0.0	8.1	0.0
Incomplete response	31.1	18.8	49.5	1.1
Incomplete	15.5	0.0	2.7	0.0
No process	10.7	54.5	2.2	25.0
No response	4.9	0.0	44.6	0.0

As can be seen in the table 1, the preferred process used in correct conception group was the *left to right process*. 100.0 % of English and Thai pupils using this process gained the correct answer. In the misconception group, 6.8 % of English pupils ignored the letters while 8.1 % of Thai pupils used the *substitution* method. Of the incomplete response group, a large number (44.6 %) of Thai pupils made no attempt. 15.5 % of English pupils made only a partial attempt.

Within the correct conception group, there are four sub-processes. *The incorrect operation process* is the responses with different operations from the question given or wrong order of operating. *The letter ignored computing process* refers to those who tried to work with coefficients only. *The left to right computing process* reflects the way of combining the first two terms and then the next term as a question set up. *The plus to minus computing process* deals with the positive term and then negative term. The most popular process pupils used to solve this problem was the *left to right computing*. This process was used among the correct conception group in both countries. For example, they showed their processes as “ $(2a-a)=1a+3a=4a$ ”, and “ $2a-a=a, a+3a=4a$ ”. The second most popular was the *plus to minus computing process*. This process was used among the correct conception group in both countries. For instance, they showed their processes as “ $2a+3a=5a-a=4a$ ”.

There are three sub-processes using within the misconception group. *The incorrect operation process* is that deals with number rather than simplifies like terms, or set up an equation. *The letter ignored computing*, which attempted to simplify unlike terms. *The substitution process*, in which a particular value is assumed and hence a numerical answer obtained. The misconception group in both countries used *the letter ignored computing process*. For example, the participants in both countries showed their processes as “ $2a-a=2$ (number ignored), $2+3a=5a$ (letter ignored)”. English pupils illustrated their process as “ $2a-a=1a+3=4$ (letter ignored), and “ $2a-a=2+3=5+a=5a$ (number ignored, letter ignored, incorrect operation)”. Thai pupils showed their processes as “ $2a-a+3a, 5a-a, a-a, a=5$ (plus, number ignored, numerical answer)” and “ $2a-a+3a=2+3=5$ (letter ignored, combined numbers appear in the expression)”. *The substitution process* was popular among Thai misconception group. They responded to the question as “ $(2 \times 1)-1+3 \times 1=(2-1)+3=1+3=4$ (substitute $a=1$)”, “ $(2 \times 2)-2+(3 \times 2)=(4-2)+5=2+5=7$ (substitute $a=2$)”, and “ $(2 \times 4)-4+(3 \times 4)=8-4+12=4+12=16$ (substitute $a=4$)”.

There are four sub-processes in the incomplete response group. *The incomplete computing*, which attempted to work it out but did not reach completion. *No process*, which gave the answer without working. *No response*, which offered no solution and no working. English pupils in this group attempted to simplify as “ $2a+3a, 5a-a$ ”, “ $2a-a=a, a+3a$ ”, and “ $3a(2a-a)$ ”. A large number of Thai pupils made no response to this item.

The results indicate that about half of English and only around a third of Thai participants had abilities to simplify like terms. This suggests there are likely to have even greater problems on the harder level of difficulty.

Level two Item 8 Simplify the expression $6+3b-c-6b-c+2$.

Level two question was designed to investigate how pupils manipulate a two variable expression. The preferred process used in correct conception group was *the grouping strategy*. 22.3 % of English pupils with of those 65.2 % gained correct answer and

4.8 % of Thai pupils with of those 88.9 % gained correct answer. They showed their works as “ $6+2+3b-6b-c-c$ ” and then simplify them. The less successful respondents tended to make the incorrect simplification of $-c-c$, which ignored the first minus sign. They simplified $c-c=0$ instead of $-c-c=-2c$. These responses indicate the error on arithmetic rather than algebra itself. In the misconception group, 7.8 % of English pupils and 13.4 % of Thai pupils showed their processes as *the letter ignored process*. For instance, they addressed the processes as “ $6+3b=9b-c=8b-6b=2b-c=1b+2=3b$ ”. Only Thai pupils used *the incorrect operation* by setting an equation and made balancing confusion. They illustrated the processes as “ $6+3-c-6-c+2$ (cancelling b), $6+3-c+c-6-c+c+2$ (balancing confused), $6+3- -6- +2$ (minus sign confused), $6+3+6+2=9+6+2=17$ ” (a numerical answer). Of the incomplete response group, 32.0 % of English pupils gave the answer without working, 70.4 % of Thai pupils made no attempt.

These results confirm the difficulties the pupils had in simplifying algebraic expressions with the negative signs.

Level three Item 14 Simplify $3p+5(p-3)-2(q-4)$

Level three question was designed to observe how pupils multiply out the brackets and simplify expression. The preferred process used in the correct conception group was *the incorrect operation process*. 29.1 % of English pupils with of those 36.7 % gained the correct answer, and 4.8 % of Thai pupils with of those 11.1 % gained the correct answer. The confusion they face was operated the negative signs such as “ $3p+5p-15-2q-8=8p-7-2q$ ”, and “ $3p+5p-15-2q-8=8p-2q-23$ ”. The first strategy gained the correct answer but the second method gained the incorrect answer. In the misconception group, 8.7 % of English pupils used *the letter ignored method*. 3.2 % of Thai pupils showed *the incorrect operation*. For instance, they showed the processes as “ $3p+5\times-3p-2\times-4q$, $3p-15p-8q$ ” and “ $3p+(5p-15)-(2q-8)$, $3p+ -10p-(-6q)$, $-7p-(-6q)$, $-7p+6q$ ”. Of the incomplete response group, 42.7 % of English and 71.0 % of Thai pupils made *no response* to this question.

The results indicate the problems pupils had with multiplying out the brackets and computing negative numbers. Some of them view “p” as “q”, and vice versa.

Level four Item 20 Multiply out the bracket and then simplify $x^2+2xy-3(xy-2x^2)$

The level four question was designed to gain insight into how pupils multiplying out the brackets and simplifying the like terms in a two variable expression with second order and brackets included. Most pupils in both countries made *no response* to this question, 46.6 % of English and 75.3 % of Thai pupils. The preferred process used in the correct conception group was *the incorrect operation process*. In both the correct conception and the misconception groups showed their works with *the incorrect process*. 17.5 % of English participants in the correct conception group used *the incorrect operation* and of those 22.2 % gained the correct answer. For instance, the correct conception group showed the processes with *the incorrect operation* as

“ $x^2+2xy-3xy-6x^2$, $7x^2+ -xy$ ”, “ $x^2+2xy-3xy-6x^2$, $-5x^2+ -xy$ ”, and “ $x^2+2xy-3xy-6x^2$, $x^2-6x^2+2xy-3xy$, $2x-12x+5xy$, $10x+5xy$ ”. The first example gained the correct answer with twice minus sign confused when multiplying out the brackets and simplifying like terms. The second example gained the wrong answer with one step of confusion with minus sign when expanding brackets. The third one indicated the confusion not only negative signs but also indices. The misconception group addressed the processes with *the incorrect operation* as “ $-x^2-4x^4$, $2y-3y=y$, $2x-4x^6-y$ ” and “ $x^2+2x\times 2y-(3x\times 3y-6x^2)$, $7x^4+(-x)\times(-y)$, $7x^4+xy$, $8x^4+y$ ”.

The less successful in solving level 4 item 20 multiply out the bracket and then simplify $x^2+2xy-3(xy-2x^2)$ confirmed their inability to view the like terms and the negative sign confused.

SUMMARY AND CONCLUSION

From its results, we can see that English and Thai pupils in the correct conception group used the same process to simplify the expressions. The misconception group, English pupils used *the letter ignored process* to solve level one, two and three problems. They made *the incorrect operation* on level four question. Thai pupils used *the substitution process* to solve the level one problem. *The letter ignored process* was used to approach the level two question. They used *the incorrect operation process* to deal with the level three and level four expressions. The main difficulties were seeking like terms, and dealing with negative signs.

The misconception not seen in the English participants’ responses which appeared in the work of Thai participants was *the incorrect operation process* “set up an equation”. This process reflects the taught experiences in the Thai school. Algebra content in the Thai school is not split up as in the English school. The simplification content was taught as part of solving equations. Substitution also was taught under the topic of solving equations as part of the process of checking the results.

The process of adding and subtracting like terms in an expression is a solid base to build on in the early stages of learning algebra with understanding. A clear understanding in this process is necessary. To help the novice, more emphasis on manipulating like terms and dealing with negative signs has to be cultivated carefully. Ignorance in this process might cause our pupils difficulty in facing higher level of algebra.

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