A STUDY OF A GENERALIZATION OF BI-HYPERIDEAL, GUASI HYPERIDEAL AND INTERIOR HYPERIDEAL OF ORDERED SEMIHYPERGOUP

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ABSTRACT

In this paper, as a further generalization of hyperideal, we introduce the notion of bi-quasi-interior hyperideal as a generalization of bi-hyperideal, quasi hyperideal, interior hyper-ideal, bi-interior hyperideal and bi-quasi hyperideal of ordered semihypergroup and study the properties of bi-quasi-interior hyperideal of ordered semihypergroup.

Keywords: bi-hyperideal, quasi hyperideal, interior hyperideal, bi-interior hyperideal, bi-quasi hyperideal, bi-quasi-interior hyperideal, regular ordered semihypergroup, simple ordered semihypergroup

1. Introduction

The first step of the development of hyperstructure theory, in particular hypergroup theory, can be traced back to the 8th Congress of Scandinavian Mathematicians in 1934, when Marty [1] introduced the concept of hypergroups, analyzed its properties and applied it to groups, rational fractions and algebraic functions. Later on, people have observed that hyperstructures have many applications to several branches of both pure and applied sciences [2,3]. In particular, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. Nowadays many scholars have studied different aspects of semihypergroups. The concept of ordering hypergroups investigated by Chvalina [22], as a special class of hypergroups and studied by him and many others. Heidari and Davvaz[16], applied the theory of hyper-strudtures to ordered semigroups and introduced the concepts of ordered semihypergroups, which is a generalization of ordersemigroups. Heiderde semihypergroups, also studied hyperideals of ordered semihyper-groups in[16].

Changphas and Davvaz introduced the concepts of bi- hyperideal and quasi hyperideals in ordered semihypergrops[20].

In 2018, M. Murali Krisna Rao[2] studies ideals in semigroup and introduced the notion of bi-quasi-interior ideal as generalization of quasi ideal, bi-ideal and interior ideal of semigroup and studies the properties of quasi-ideal, bi-ideal and interior ideal of semigroup, simple semigroup and regular semigroup.

In this paper, as a further generalization of hyperideal, we introduce the notion of bi-quasi-interior hyperideal as a generalization of bi-hyperideal, quasi hyperideal, interior hyperideal, bi-interior hyperideal and bi-quasi hyperideal, of ordered semihypergroup and study the properties of bi-quasi-interior hyperideal of ordered semihypergroup and some characterizations of bi-quasi-interior hyperideal of ordered semihypergroup, regular ordered semihypergroup and simple ordered semihypergroup.

2. Preliminaries and basic definitions

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

The concept of ordering hypergroups investigated by Chvalina[3] as a special of hypergroups and studied by himand many othetrs. In [5] Heidari and Davvaz studied a semihypergroup $(M,\circ)$ besides a binary relation $\leq$ where $\leq$ is a partial order relation such that satisfies the monotone condition.

Let M be a nonempty set. A mapping $\circ : M \times M \to P^*(M)$ where $P^*(M)$ denotes the family of all nonempty subsets of M, is called a hyperoperation on M. The couple $(M,\circ)$ is called a hypergroupoid. In the above definition, if A and B are two nonempty subsets of S and $x \in S$, then we denote $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$. A hypergroupoid $(S,\circ)$ is called a semihypergroup if for every x,y,z in S, $x \circ (y \circ z) = (x \circ y) \circ z$. That is, $\bigcup_{u \in y \circ u} x \circ u = \bigcup_{v \in x \circ y} v \circ z$. In [5], Heidari and Davvaz studied a semihypergroup $(M,\circ)$ besides a binary relation $\leq$, where $\leq$ is a partial order relation such that satisfies the monotone condition. Indeed, an ordered semihypergroup $(S,\circ,\leq)$ is a semihypergroup $(M,\circ)$ together with a partial order $\leq$ that is compatible with the hyperoperation on, meaning that for any x,y,z in M,
$x \leq y \Rightarrow z \circ x \leq z \circ y$ and $x \circ z \leq y \circ z$. Here, $z \circ x \leq z \circ y$ means for any $a \in z \circ x$ there exists $b \in z \circ y$ such that $a \leq b$. The case $x \circ z \leq y \circ z$ is defined similarly.

Note that the concept of ordered semihypergroups is a generalization of the concept of ordered semi-groups [1],[6],[7] Indeed, every ordered semigroup is an of ordered semihypergroups. For a nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ we write $(A] = \{x \in M \mid x \leq a \text{ for some } x \in A\}$.

The following is easy to sec for nonempty subsets A, B of an ordered semihypergroup $(M,\circ,\leq)$; (i) $A \subseteq (A]$. (ii) $A \subseteq B \Rightarrow (A] \subseteq (B]$. (iii) $(A] \circ (B] \subseteq (A \circ B]$. (iv) $((A] \circ (B]] = (A \circ B]$. (v) $(A] \cup (B] = (A \cup B]$.

Definition 1[12] An ordered semihypergroup $(M,\circ,\leq)$ is called an ordered hypergroup if $(M,\circ)$ is a hypergroup. In fact, for all a in M, We have $a \circ M = M \circ a = M$.

Definition 2 A nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ is called idempotent if $A = (A \circ A]$.

Definition 3[12] A nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ is called a subsemihypergroup of S if $A\circ A\subseteq A$.

Definition 4[12] A nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ is called a left (respectively, right) hyperideal of M if it satisfies the following condition:

(i) $M\circ A\subseteq A$ (respectively, $A\circ M\subseteq A$). (ii) If $x\in A$ and $y\in M$ such that $y\leq x$ then $y\in A$, if A both a left and right hypergroup of M, then it is called a two-sided hyperideal of M, or a hyperideal of M.

Example. 2.1 We have $M=\{a,b,c,d\}$ and $B=\{a,b,c\}$. Define operation $\circ$ on M by the following table;

$\circ$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	a
b	a	$\{a,b\}$	$\{a,c\}$	a
c	a	$\{a,b\}$	$\{a,c\}$	a
d	a	$\{a,b\}$	$\{a,c\}$	a

$\leq=\{a,a\},\{b,b\},\{c,c\},\{d,d\},\{b,a\},\{c,a\}$. Then B is a hyperideal of M.

Definition 5[11] An ordered semihypergroup $(M,\circ,\leq)$ is called regular if for every $a\in M$ there exists $x\in M$ such that $a\leq a\circ x\circ a$. If it satisfies the following conditions:

(i) $A\subseteq(A\circ M\circ A], \forall A\subseteq M$. (ii) $a\in(a\circ M\circ a], \forall a\in M$.

Definition 6[11] A subsemihypergroup A of an ordered semihypergroup $(M,\circ,\leq)$ is called an interior hyperideal of M if it satisfies the following condition:

(i) $M\circ A\circ M\subseteq A$. (ii) If $x\in A$ and $y\in M$ such that $y\leq x$, than $y\in A$.

Definition 7[12] A nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ is called quasi-hyperideal of M if it satisfies the following condition:

(i) $(M\circ A)\cap(A\circ M)\subseteq A$. (ii) If $x\in A$ and $y\in M$ such that $y\leq x$, than $y\in A$.

Definition 8[12] A nonempty subset A of an ordered semihypergroup $(M,\circ,\leq)$ is called a bi-hyperideal of M if it satisfies the following condition:

(i) $A\circ M\circ A\subseteq A$. (ii) If $x\in A$ and $y\in M$ such that $y\leq x$, than $y\in A$.

Definition 9 A subsemihypergroup A of an ordered semihypergroup $(M,\circ,\leq)$ is called a bi–interior hyperideal of S if it satisfies following condition:

(i) $(M\circ A\circ M)\cap(A\circ M\circ A)\subseteq A$. (ii) If $x\in A$ and $y\in M$ such that $y\leq x$, than $y\in A$.

Definition 10 A subsemihypergroup B of an ordered semihypergroup $(M,\circ,\leq)$ is called a left (right) bi-quasi hyperideal of M if it satisfies following condition:

(i) $(M\circ A)\cap(A\circ M\circ A)\subseteq A$, $(A\circ M)\cap(A\circ M\circ A)\subseteq A$. (ii) If $x\in A$ and $y\in M$ such that $y\leq x$, than $y\in A$.

And, a non-empty subset A of M called a bi-quasi hyperideal of M if it both a left bi-quasi hyperideal and right bi-quasi hyperideal of M.

3. Research Methodology

In this section we introduce the notion of bi-quasi-interior hyperideal as a generalization of bi-hyperideal, quasi-hyperideal and interior hyperideal of ordered semihypergroup and study the properties of bi-quasi-interior hyperideal of ordered semihypergroup.

Definition 3.1 A non-empty subset B of ordered semihypergroup $(M, \circ, \le)$ is called a bi-quasi-interior hyperideal of S if it satisfies the following condition;

(i) $(B \circ M \circ B \circ M \circ B] \subseteq B$. (ii) If $x \in B$ and $y \in M$ such that $y \le x$, than $y \in B$.

Example. 3.1 We have $M = \{a, b, c, d, f\}$ and $B = \{a,\}$. Define operation $\circ$ on M by the following table;

$\circ$	a	b	c	d	f
a	a	a	a	a	a
b	a	$\{a,b\}$	a	$\{a,d\}$	a
c	a	$\{a,f\}$	$\{a,c\}$	$\{a,c\}$	$\{a,f\}$
d	a	$\{a,b\}$	$\{a,d\}$	$\{a,d\}$	$\{a,b\}$
f	a	$\{a,f\}$	a	$\{a,c\}$	a

$\le = \{a,a\}\{b,b\}\{c,c\}\{d,d\}\{f,f\}\{a,b\}\{a,c\}\{a,d\}\{a,f\}$. Then B is a bi-quasi-interior hyperideal of M.

Theorem 3.1 Every bi-hyperideal of ordered semihypergroup M is a bi-quasi-interior hyperideal of ordered semihypergroup M.

Proof. Let B be bi-hyperideal of ordered semihypergroup M. Then $B \circ M \circ B \subseteq B$. Therefore $(B \circ M \circ B \circ M \circ B] \subseteq (B \circ M \circ B] \subseteq (B] = B$. Hence $(B \circ M \circ B \circ M \circ B] \subseteq B$. If $x \in B$ and $y \in M$ such that $y \le x$, we have $y \in B$. Hence B is bi-quasi-interior hyperideal of M.

Theorem 3.2 Every interior hyperideal of ordered semihypergroup M is a bi-quasi-interior hyperideal of ordered semihypergroup M.

Theorem 3.3 Let M be an ordered semihypergroup. Every left hyperideal is a bi-quasi-interior of M.

Corollary 3.1 Let M be an ordered semihypergroup. Every right hyperideal is a bi-quasi-interior hyperideal of M.

Corollary 3.2 Let M be an ordered semihypergroup. Every hyperideal is a bi-quasi-interior hyperideal of M.

Theorem 3.4 Let M be an ordered semihypergroup. Every quasi hyperideal is a bi-quasi-interior hyperideal of M.

Theorem 3.5 Let M be an ordered semihypergroup. Intersection of a right hyperideal and a left hyperideal of M is a bi-quasi-interior hyperideal of M.

Theorem 3.6 Let M be an ordered semihypergroup. If L is a left hyperideal and R is a right hyperideal of ordered semihypergroup M and $B = (R \circ L]$ then B is a bi-quasi-interior hyperideal of M.

Theorem 3.7 Let M be an ordered semihypergroup. If B is a bi-quasi-interior hyperideal and T is a non-empty subset of M such that $(B \circ T]$ is a subsemihypergroup of M. Then $(B \circ T]$ is a bi-quasi-interior hyperideal of M.

Theorem 3.8 The intersection of a bi-quasi-interior hyperideal B of ordered semihypergroup M and a right hyperideal A of M is always bi-quasi-interior hyperideal of M.

Theorem 3.9 Let A and C be a bi-quasi-interior hyperideal of ordered semihypergroup M and $B = (A \circ C]$. If $C \circ C = C$ then B is bi-quasi-interior hyperideal of M.

Proof. Let A and C be a bi-quasi-interior hyperideal of ordered semihypergroup M and $B = (A \circ C]$. We have $B \circ B = (A \circ C] \circ (A \circ C] \subseteq (A \circ C \circ A \circ C] \subseteq (A \circ C \circ C \circ A \circ C] \subseteq (A \circ C \circ C \circ C \circ A \circ C]$ $\subseteq (A \circ C \circ M \circ C \circ M \circ C] \subseteq (A \circ C] = B$. Thus $B = (A \circ C]$ is a subsemihypergroup of M. Now, $(B \circ M \circ B \circ M \circ B] = ((A \circ C] \circ M \circ (A \circ C] \circ M \circ (A \circ C]] \subseteq ((A \circ C \circ M \circ A \circ C \circ M \circ A \circ C]]$ $\subseteq (A \circ C \circ M \circ A \circ C \circ M \circ A \circ C] \subseteq (A \circ M \circ A \circ M \circ A \circ C] \subseteq (A \circ C] = B$. Let $x \in (A \circ C]$ and $y \in M$ be such that $y \leq x$. Since $y \in (A \circ C]$, we have $x \leq z$ for some $z \in (A \circ C]$. Then $y \leq x \leq z$, which implies that $y \in A \circ C$. Hence B is a bi-quasi-interior hyperideal of M.

Corollary 3.3 Let A and C be a bi-quasi-interior hyperideal of ordered semihypergroup M and $B = (C \circ A]$. If $C \circ C = C$ then B is bi-quasi-interior hyperideal of M.

Theorem 3.10 Let A and C be a subsemihypergroup of M and $B = (A \circ C]$. If A is a left hyperideal then B is bi-quasi-interior hyperideal of M.

Corollary 3.4 Let A and C be a subsemihypergroup of M and $B = (A \circ C]$. If A is a right hyperideal then B is bi-quasi-interior hyperideal of M.

Theorem 3.11 B is bi-quasi-interior hyperideal of ordered semihypergroup M if and only if B is a left hyperideal of some right hyperideal of an ordered semihypergroup.

Proof. Assume that B is bi-quasi-interior hyperideal of ordered semihypergroup M. Then $(B \circ M \circ B \circ M \circ B] \subseteq B$. It follow that $(B \circ M \circ B \circ M] \circ M = ((B \circ M \circ B \circ M] \circ (M]$ $\subseteq (B \circ M \circ B \circ M \circ M] \subseteq (B \circ M \circ B \circ M]$. Let $x \in (B \circ M \circ B \circ M]$ and $y \in M$ be such that $y \leq x$. Since $x \in (B \circ M \circ B \circ M]$, we have $x \leq z$ for some $z \in B \circ M \circ B \circ M$. Then $y \leq x \leq z$, which implies that $y \in (B \circ M \circ B \circ M]$. Therefore $(B \circ M \circ B \circ M]$ is a right hyperideal of M. Then $(B \circ M \circ B \circ M] \circ B = ((B \circ M \circ B \circ M] \circ (B] \subseteq (B \circ M \circ B \circ M] \subseteq (B] = B$. Thus B is a left hyperideal of some right ideal of M.

Conversely, assume that B is a left hyperideal of right hyperideal R of M. We have $R \circ M \subseteq R$ and $R \circ B \subseteq B$. Then $(B \circ M \circ B \circ M \circ B] \subseteq ((B \circ M \circ B \circ M] \circ B] \subseteq (B] = B$. Hence $(B \circ M \circ B \circ M \circ B] \subseteq B$. If $x \in B$ and $y \in M$ such that $y \leq x$, then $y \in B$. Hence B is bi-quasi-interior hyperideal of M.

Corollary 3.5 B is bi-quasi-interior hyperideal of an ordered semihypergroup M if and only if B is a right hyperideal of some left hyperideal of an ordered semihypergroup.

Theorem 3.12 If B is bi-quasi-interior hyperideal of an ordered semihypergroup M, T is a subsemihypergroup of M and $T \subseteq B$ then $(B \circ T]$ is a bi-quasi-interior hyperideal of M.

Theorem 3.13 Let B be a bi- hyperideal of an ordered semihypergroup M and I be an interior hyperideal of M. Then $B \cap I$ is a bi-quasi-interior hyperideal of M.

Theorem 3.14 Let M be an ordered semihypergroup. If $M = (M \circ a]$ for all $a \in M$. Then every bi-quasi-interior hyperideal of M is a quasi hyperideal of M.

Theorem 3.15 The intersection of bi-quasi-interior hyperideal $\{B_l \mid l \in A\}$ of an ordered semihypergroup M. is bi-quasi-interior hyperideal of M.

Proof. Let $B = \bigcap_{l \in A} B_l$. Then B is a subsemihypergroup of M. Since B_l is a bi-quasi-interior hyperideal of M, we get $(B_l \circ M \circ B_l \circ M \circ B_l] \subseteq B_l$ for all $l \in A$. Hence $((\bigcap B_l) \circ M \circ (\bigcap B_l) \circ M \circ (\bigcap B_l)] \subseteq \bigcap B_l = B$. If $x \in B$ and $y \in M$ such that $y \leq x$, then $y \in B$ Therefore B is a bi-quasi-interior hyperideal of M.

Theorem 3.16 Let B be an interior hyperideal of ordered semihypergroup M, $A \subseteq B$ and A be an idempotent. Then $(A \circ B]$ is a bi-quasi-interior hyperideal of M.

Proof. Let B be an interior hyperideal of ordered semihypergroup M, Obviously $(A \circ B]$ is a subsemihypergroup of M. By considering $(A \circ M] \circ M \circ (A \circ M] \subseteq (A \circ M] \circ (M] \circ (A \circ M] \subseteq (A \circ M \circ M \circ A \circ M] \subseteq (A \circ M]$. Let $x \in (A \circ M]$ and $y \in M$ be such that $y \leq x$. Since $x \in (A \circ M]$ we have $x \leq z$ for some $z \in A \circ M$. Then $y \leq x \leq z$, which implies that $y \in (A \circ M]$. Hence $(A \circ M]$ is a bi- hyperideal of M. Suppose that $x \in B \cap (A \circ M]$, then $x \in B$ and $x \in (A \circ M]$. Since $x \in (A \circ M]$, we have $x \leq e \circ y$ for some $e \circ y \subseteq (A \circ M]$. Then $x \leq e \circ y \subseteq (A \circ M] = ((A \circ A] \circ M] = ((((A \circ A] \circ A] \circ M]$ $= (((A \circ A] \circ (A \circ A]] \circ M] \subseteq (A \circ A \circ A \circ A \circ M] \subseteq (A \circ B \circ M \circ B \circ M] \subseteq (A \circ B \circ B] \subseteq (A \circ B]$. Hence $x \in (A \circ B]$. Therefore $B \cap (A \circ M] \subseteq (A \circ B]$. Since $(A \circ B] \subseteq (B] = B$ and $(A \circ B] \subseteq (A \circ M]$ we have $(A \circ B] \subseteq B \cap (A \circ M]$. This show that $B \cap (A \circ M] = (A \circ B]$. By theorem 3.13 we get that $(A \circ B]$ is a bi-quasi interior hyperideal of M.

Corollary 3.6 Let M be ordered semihypergroup and B be an idempotent. Then $(B \circ M]$ and $(M \circ B]$ are bi-quasi-interior hyperideal of M.

Theorem 3.17 If B be a left bi-quasi hyperideal of ordered semihypergroup M, then B is a bi-quasi-interior hyperideal of M.

Proof. Assume that B is a left bi-quasi-interior hyperideal of ordered semihypergroup M. Now, $(B \circ M \circ B \circ M \circ B] \subseteq (M \circ B]$ and $(B \circ M \circ B \circ M \circ B] \subseteq (B \circ M \circ B]$. We have $(B \circ M \circ B \circ M \circ B] \subseteq (M \circ B] \cap (B \circ M \circ B] \subseteq B$. Clearly, if $x \in B$ and $y \in M$ such that $y \leq x$, then $y \in B$. Hence B is a bi-quasi-interior hyperideal of M.

Corollary 3.7 If B be a right bi-quasi hyperideal of ordered semihypergroup M, then B is a bi-quasi-interior hyperideal of M.

Corollary 3.8 If B be a bi-quasi hyperideal of ordered semihypergroup M, then B is a bi-quasi-interior hyperideal of M.

Theorem 3.18 If B be a bi-interior hyperideal of ordered semihypergroup M, then B is a bi-quasi-interior hyperideal of M.

Proof. Suppose that B is a bi-interior hyperideal of ordered semihypergroup M. We see that $(B \circ M \circ B \circ M \circ B] \subseteq (M \circ B \circ M]$ and $(B \circ M \circ B \circ M \circ B] \subseteq (B \circ M \circ B]$, we obtain $(B \circ M \circ B \circ M \circ B] \subseteq (M \circ B \circ M] \cap (B \circ M \circ B] \subseteq B$. Clearly, if $x \in B$ and $y \in M$ such that $y \leq x$, then $y \in B$. This proves that B is a bi-quasi-interior hyperideal of M.

We introduce the notion of bi-quasi-interior simple ordered semihypergroup and characterize the bi-quasi-interior simple ordered semihypergroup using bi-quasi-interior hyperideal of ordered semihypergroup and study the properties of minimal bi-quasi-interior of ordered semihypergroup .

Definition 3.2 An ordered semihypergroup $(M,\circ,\leq)$ is said to be bi-quasi-interior simple ordered semihypergroup if M has no bi-quasi-interior hyperideals other than M itself.

Theorem 3.19 Let M be a simple ordered semihypergroup. Every bi-quasi-interior hyperideal is a bi-hyperideal of M.

Theorem 3.20 Let M be an ordered semihypergroup. Then M is a bi-quasi-interior simple ordered semihypergroup if and only if $(a\circ M\circ a\circ M\circ a]=M$, for all $a\in M$.

Proof. Suppose that M is a bi-quasi-interior simple ordered semihypergroup and $a\in M$. Now,

$x\in((a\circ M\circ a\circ M\circ a]\circ M\circ(a\circ M\circ a\circ M\circ a]\circ M\circ(a\circ M\circ a\circ M\circ a]]$

$\subseteq(a\circ M\circ a\circ M\circ a\circ M\circ a\circ M\circ a\circ M\circ a\circ M\circ a\circ M\circ a\circ M\circ a]\subseteq(a\circ M\circ a\circ M\circ a]$. Let $x\in(a\circ M\circ a\circ M\circ a]$ and $y\in M$ be such that $y\leq x$. Since $x\in(a\circ M\circ a\circ M\circ a]$, we have $x\leq z$ for some $z\in a\circ M\circ a\circ M\circ a$. Then $y\leq x\leq z$, which implies that $y\in(a\circ M\circ a\circ M\circ a]$. Hence $(a\circ M\circ a\circ M\circ a]$ is a bi-quasi-interior hyperideal of M. Since M is bi-quasi-interior simple, we have $(a\circ M\circ a\circ M\circ a]=M$.

Conversely, suppose $(a\circ M\circ a\circ M\circ a]=M$, for all $a\in M$. Let B be a bi-quasi-interior hyperideal of ordered semihypergroup M and $a\in B$. Then, $M=(a\circ M\circ a\circ M\circ a]\subseteq(B\circ M\circ B\circ M\circ B]$ $\subseteq B$. So $M=B$, which implies that M is a bi-quasi-interior simple ordered semihypergroup.

Definition 3.3[10] An ordered semihypergroup $(M,\circ,\leq)$ is said to be left (respectively, right) simple if it does not contain proper left (respectively, right) hyperideals.

Theorem 3.21 If ordered semihypergroup M is left simple ordered semihypergroup then every bi-quasi-interior hyperideal of M is a right hyperideal of M.

Proof. Let B be a bi-quasi-interior hyperideal of left ordered semihypergroup. It is easy to see that $(M\circ B]$ is a left hyperideal of M and $(M\circ B]\subseteq M$. Let $x\in(M\circ B]$ and $y\in M$ be such that $y\leq x$. Since $x\in(M\circ B]$, we have $x\leq z$ for some $z\in M\circ B$. Then $y\leq x\leq z$, which implies that $y\in(M\circ B]$. Therefore $(M\circ B]=M$. Since $B\circ M=B\circ(M\circ B]=B\circ((M\circ B]\circ B]\subseteq B\circ((M\circ B]\circ M]$ $\subseteq B\circ((M\circ B]\circ(M\circ B]]\subseteq(B\circ M\circ B\circ M\circ B]\subseteq B$. So $B\circ M\subseteq B$. Clearly $(B]=B$. Hence every bi-quasi-interior hyperideal is a right hyperideal of M.

Corollary 3.9 If ordered semihypergroup M is right simple ordered semihypergroup then every bi-quasi-interior hyperideal of M is a left hyperideal of M.

Corollary 3.10 Every bi-quasi-interior hyperideal of left and right simple ordered semihypergroup M is a hyperideal of M.

Theorem 3.22 Let M be an ordered semihypergroup and B be a bi-quasi-interior hyper-ideal of M. Then B is minimal bi-quasi-interior hyperideal of M if and only if B is a bi-quasi-interior simple ordered semihypergroup.

Theorem 3.23 Let M be an ordered semihypergroup and $B = (R \circ L]$, where L and R are minimal left hyperideal and right hyperideal of M respectively. Then B is a minimal bi-quasi-interior hyperideal of M.

Theorem 3.24 Let M be a regular ordered semihypergroup. Then every interior hyperideal of M is a hyper-ideal of M.

Proof. Let B be a interior hyperideal of M by theorem 3.2, we have B is a bi-quasi-interior hyperideal of M. Then $(BMBMB] \subseteq B$. Since M is regular, we have $B \circ M \subseteq (B \circ M \circ B] \circ M \subseteq (B \circ M \circ B \circ M]$ $\subseteq ((B \circ M \circ (B \circ M \circ B] \circ M] \subseteq (B \circ M \circ B \circ M \circ B] \circ ((B \circ M \circ B \circ M \circ (B \circ M \circ B] \circ M]$ $\subseteq (B \circ M \circ B \circ M \circ B \circ M \circ B \circ M] \subseteq (B \circ M \circ B \circ M \circ B] \subseteq B$. and $M \circ B \subseteq M \circ (B \circ M \circ B]$ $\subseteq (M \circ B \circ M \circ B] \subseteq (M \circ B \circ M \circ B \circ M \circ B] \subseteq (M \circ B \circ M \circ B \circ M \circ B \circ M \circ B]$ $\subseteq B \circ (M \circ B \circ M \circ B] \subseteq B$. Clearly, it $x \in B$ and $y \in M$ such that $y \le x$, then $y \in B$. Hence the theorem is proved.

Theorem 3.25 M is regular ordered semihypergroup if and only if $(R \circ L] = R \cap L$ for any right hyperideal R and left hyperideal L of ordered semihypergroup M.

Theorem 3.26 Let M be a regular ordered semihypergroup. Then B is bi-quasi-interior hyperideal of M if and only if $(B \circ M \circ B \circ M \circ B] = B$, for all bi-quasi-interior hyper-ideal of M.

Proof. Suppose that M is a regular ordered semihypergroup, B is bi-quasi-interior hyperideal of M. and $x \in B$. Then $(B \circ M \circ B \circ M \circ B] \subseteq B$ and there exists $y \in M$ such that $x \le x \circ y \circ x \le x \circ y \circ x \circ y \circ x$ $\in B \circ M \circ B \circ M \circ B$. Thus $x \in (B \circ M \circ B \circ M \circ B]$. So $B \subseteq (B \circ M \circ B \circ M \circ B]$. Hence $(B \circ M \circ B \circ M \circ B] = B$.

Conversely, suppose $(B \circ M \circ B \circ M \circ B] = B$, for all B is bi-quasi-interior hyperideal of M. Let $B = R \cap L$, where R is a right hyperideal and is a left hyperideal of M. By theorem 3.5, we have $R \cap L$ is bi-quasi-interior hyperideal of M. Therefore $R \cap L = ((R \cap L] \circ M \circ (R \cap L] \circ M \circ (R \cap L]]$ $\subseteq (R \circ M \circ L \circ M \circ L] \subseteq (R \circ L]$. Since $(R \circ L] \subseteq (L] = L$ and $(R \circ L] \subseteq (R] = R$, Hence $R \cap L =$ $(R \circ L]$. This show that $R \cap L = (R \circ L]$. Hence by theorem 3.25, M is a regular ordered semihypergroup.

Theorem 3.27 Let B a subsemihypergroup of regular ordered semihypergroup M then B can be represented as $B = (R \circ L]$, where R is a right hyperideal and L is a Left hyperideal of M if and only if B is a bi-quasi-interior hyperideal of M.

Proof. Suppose that $B = (R \circ L]$, where R is a right hyperideal and L is a left hyperideal of M. The following holds $(B \circ M \circ B \circ M \circ B] = ((R \circ L] \circ M \circ (R \circ L] \circ M \circ (R \circ L]]$ $\subseteq (R \circ L \circ M \circ R \circ L \circ M \circ R \circ L] \subseteq (R \circ L] = B$ Let $x \in (R \circ L]$ and $y \in M$ be such that $y \le x$. Since $x \in (R \circ L]$, we have $x \le z$ for some $z \in R \circ L$. Then $y \le x \le z$, which implies that $y \in (R \circ L]$. Thus B is a bi-quasi-interior hyperideal of ordered semigroup M.

Conversely, suppose That B is a bi-quasi-interior hyperideal of regular ordered semihypergroup M. By Theorem 3.26 we have $(B \circ M \circ B \circ M \circ B] = B$. Setting $R = (B \circ M]$ and $L = (M \circ B]$. We have $R = (B \circ M]$ is a right hyperideal of M and $L = (M \circ B]$ is a left hyperideal of M. Consider $(B \circ M] \cap (M \circ B] = (B \circ M](M \circ B] \subseteq (B \circ M \circ M \circ B] \subseteq (B \circ M \circ B \circ M \circ B] = B$. Hence $R \cap L = (B \circ M] \cap (M \circ B] \subseteq B$. Since $B \subseteq (B \circ M] = R$ and $B \subseteq (M \circ B] = L$ we have $B \subseteq R \cap L$. It is clear that $B = B \cap L = (R \circ L]$. Thus B can represented as $(R \circ L]$, where R is a right hyperideal and L is a left hyperideal of M. Hence the theorem is proved.

Theorem 3.28 M is regular ordered semihypergroup if and only if $B \cap I \cap L \subseteq (B \circ I \circ L]$, for any bi-quasi-interior hyperideal B, hyperideal I, and left hyperideal of M.

Proof. Suppose that M be a regular ordered semihypergroup, B, I and L are bi-quasi-interior hyperideal, hyperideal and left hyperideal of M respectively. Let $a \in B \cap I \cap L$. Since M is a regular, we have $a \in (a \circ M \circ a]$. So $a \leq a \circ y \circ a$ for same $y \in M$. Then $a \leq a \circ y \circ a \leq a \circ y \circ a \circ y \circ a$ $\in B \circ M \circ I \circ M \circ L \subseteq B \circ I \circ L$. Thus $a \in (B \circ I \circ L]$. It is clear that $B \cap I \cap L \subseteq (B \circ I \circ L]$.

Conversely, suppose that $B \cap I \cap L \subseteq (B \circ I \circ L]$, for any bi-quasi-interior hyperideal B, hyperideal I and left hyperideal L of M. Let R be a right hyperideal and L be a left hyperideal of M. Then by assumption $R \cap L = R \cap M \cap L \subseteq (R \circ M \circ L] \subseteq (R \circ L]$. We have $(R \circ L] \subseteq (R \circ M] \subseteq (R] = R$ and $(R \circ L] \subseteq (M \circ L] \subseteq (L] = L$. Therefore $(R \circ L] \subseteq R \cap L$. Hence the theorem is proved.

4. Research Findings

As a further generalization of hyperideals, we introduced the notion of bi-quasi-interior hyperideal of ordered semihypergroup as a generalization of hyperideal, left hyperideal, right hyperideal, bi- hyperideal, quasi hyperideal and interior hyperideal of ordered semihypergroup and studied some of their properties. We proved every bi-quasi hyperideal of ordered semihypergroup and bi-interior hyperideal of ordered semihypergroup are bi-quasi-interior hyperideals and studied some of the properties of bi-interior hyperideals of ordered semihypergroup. In continuity of this paper, we study prime bi-quasi-interior hyperideals, prime, maximal and minimal bi-quasi-interior hyperideals of ordered semihypergroup.

5. Acknowledgement

This research was well done because of helping from advisor Wichayaporn Jantanan for suggestion and offering some opinions to profit of this research. And help to fixed problems during the proceed. And also we would like to say thank you for friends in our majoring from Buriram Rajabhat University for encouraging and help in this research.

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