แบบจำลองเชิงฮอโลกราฟีของดาวมัลติควาร์ก

HOLOGRAPHIC MODEL OF A MULTIQUARK STAR

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บทคัดย่อ

บทความในงานวิจัยนี้ศึกษาเกี่ยวกับเสถียรภาพของดาวที่มีความหนาแน่นสูงและมีอุณหภูมิคงตัวโดยใช้หลักการฮอโลกราฟีตามแนวทางของแบบจำลองซาไก-ซูกิโมโตะ ดาวที่ศึกษานั้นได้ถูกสมมติให้มีขนาดใหญ่ มีสมมาตรทรงกลม และสสารภายในดาวอยู่ในสภาพที่ไม่มีการกักขังเนื่องจากดาวมีความดันสูงและอุณหภูมิคงตัวที่สูงพอประมาณทั่วทั้งดาว ภายใต้ข้อสันนิษฐานที่ว่าในสภาวะซึ่งไม่มีการกักขังนั้น กลูออนมีอิสระที่จะเคลื่อนที่ไปได้ทั่วทั้งดาวแต่ควาร์กนั้นอาจอยู่ในสถานะอันใหม่ซึ่งเราเรียกว่าสถานะมัลติควาร์ก โดยสถานะนี้ไม่จำเป็นต้องมีเงื่อนไขที่ว่าเลขควอนตัมเชิงรงค์ต้องรวมกันแล้วไม่มีสี ในท้ายที่สุดเราใช้สมการโทลมัน-โอเพนไฮเมอร์-วอลคอฟ และสมการสถานะของดาวมัลติควาร์กที่ได้จาก แบบจำลองซาไก-ซูกิโมโตะ ทำการประมาณค่ามวลของดาวมัลติควาร์ก

คำสำคัญ: มัลติควาร์ก, ดาวมัลติควาร์ก, หลักการฮอโล-กราฟี, แบบจำลองซาไก-ซูกิโมโตะ, สมการโทลมัน-โอเพนไฮมเมอร์-วอลคอฟ

ABSTRACT

Stability of a hypothetical dense warm star was studied through a holographic principle, particularly Sakai-Sugimoto model. The massive star was assumed to be spherical symmetric and entirely in a deconfined phase due to the high density and a uniform moderate temperature throughout the star. It is assumed that, in the gluon-deconfined phase, gluons are free to propagate but quarks could form a new bound state without color singlet condition therefore the state is called a multiquark state. Finally, using the Tolman-Oppenheimer-Volkoff (TOV) equation and the equation of states of the multiquarks from the Sakai-Sugimoto model, we established the mass limits of the multiquark star and explored certain hydrodynamic properties of the multiquark matter within the star.

Keywords: multiquark, multiquark star, holographic principle, Sakai-Sugimoto model, TOV equation

INTRODUCTION

The failure to detect a free quark in the laboratory is strong evidence that the coupling constant for strong interaction becomes nonperturbatively large at low energy scale. It is said that quarks and gluons are confined inside hadrons with colorless condition at low energies. When the energy or temperature scale increase, the coupling of the strong interaction becomes weaker and eventually we expect the deconfinement to occur. Additionally if the quarks and gluons are compressed by extremely high pressure, quarks could interact with neighboring quarks and gluons equally and become deconfined from the mesonic or baryonic bound state. It is possible to have a situation that nuclear matter is under a very high pressure and the temperature is in between the gluon deconfinement and chiral symmetry restoration. In such case, the coupling could still be strong despite of the deconfinement so that a perturbative method is unreliable.

Fortunately, the evolution of the holographic principle and AdS/CFT correspondence (Maldacena, 1998) in the context of superstring theory provides us with a new method to investigate the physics of strongly coupled nuclear matter both in the low energy regime and in the energy scale close to the deconfinement temperature. A holographic model of mesons was proposed that quark and antiquark potential in meson can be calculated from Nambu-Goto action of the string in the bulk at zero and finite temperature (Maldacena, 1998; Rey et al., 1998; Rey & Yee, 2001). Subsequently, baryons are proposed to be a D-brane wrapping internal subspace of background spacetime with N_c strings stretching out to the boundary of AdS space with a color singlet condition (Witten, 1998; Gross & Ooguri, 1998).

A more generalized holographic model for deconfined color bound states of quarks, called multiquark, can be studied in the Sakai-Sugimoto (SS) model (Sakai & Sugimoto, 2005a, 2005b). The key feature of this model is a possibility of chiral symmetry breaking which is similar to the real QCD. It was found that the multiquark phase is thermodynamically stable and preferred over the other phaes in the gluon-deconfined plasma provided that the density is sufficiently large (Burikham et al., 2009).

The situation of high pressure and moderate temperature could coexist within specific classes of compact stars and it is fascinating to inquire into thermodynamic properties and the stability of this kind of dense stars. In this paper, the hypothetical multiquark star, obeying the equation of state derived from the holographic multiquarks in the SS model, is explored. With the power-law approximation of the equations of state, we investigate its gravitational stability using the Tolman-Oppenheimer-Volkov equation (Tolman, 1939; Oppenheimer & Volkov, 1939). The mass, density and pressure distributions are determined numerically. The mass-radius relation and the mass limit are also discussed.

Thermodynamic properties of multiquark

According to the SS model, the multiquark state can be described by a configuration consisting of N_f D8-branes and N_f anti-D8-branes joined together with D4 vertex and radial strings stretching down from the D4 vertex as shown in Fig.1(c).

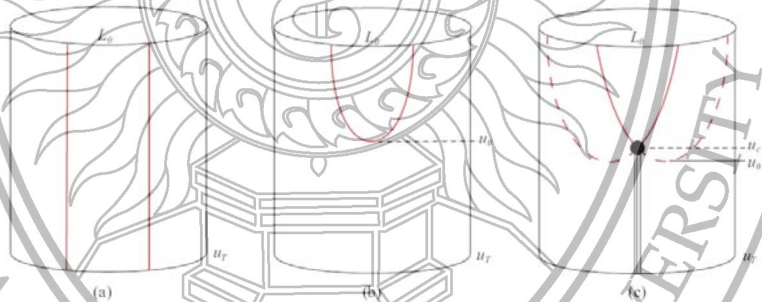

FIGURE 1. Different configurations of (a) χ_S-QGP (b) vacuum (c) exotic multiquark phase (Burikham et al., 2010).

The relations between pressure (P) and number density (d) of multiquark in deconfined background at a relatively small finite temperature are given as the following:

For small d limit,

$$P \simeq \frac{\alpha_0}{2} d^2 - \frac{3\beta_0(n_s)}{4} d^4 \quad (1)$$

where α_0 is a constant and $\beta_0(n_s)$ is a function of number density of radial strings n_s.

For large *d* limit,

$$P \approx \frac{2}{35}\left(\frac{\Gamma(\frac{1}{5})\Gamma(\frac{3}{10})}{\Gamma(\frac{1}{2})}\right)d^{7/5}. \qquad (2)$$

The relations between pressure and number density of multiquark in deconfined phase at finite temperature can be summarized into the graph below.

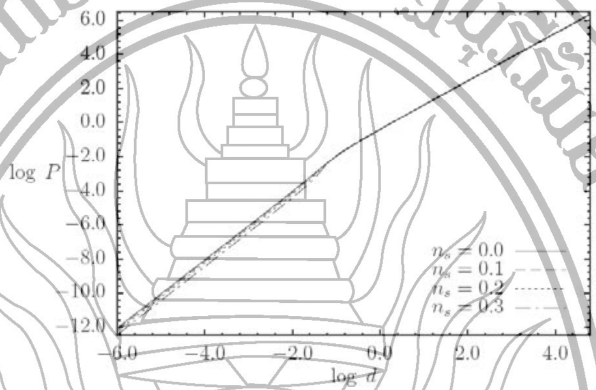

FIGURE 2. Pressure and density in logarithmic scale at T=0.03 (Burikham et al., 2010).

The pressure is approximately independent of the temperature in our consideration range and we therefore present only the plots at T = 0.03. The transition from small to large d was clearly seen in the logarithmic-scale plots around $d_c \approx$ 0.072. Interestingly, existence of color charges in the multiquark bound states reduces the pressure of the system.

Gravitational stability of a multiquark star

In this section, we will consider a hypothetical multiquark star containing only the multiquark matter with uniform constant temperature. The relations between pressure and density will be used directly from the holographic model as the equations of state of the quasi-particles. Since the pressure and density have very small temperature dependence for the range of temperatures under consideration, the results are valid generically.

A study into the gravitational stability of a spherically symmetric dense star can be performed using the Tolman-Oppenheimer-Volkov equation. It is known that the spherically symmetric dense star has metric in a form

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2 . \qquad (3)$$

After substituting into the Einstein field equation, we obtain the following relations,

$$B(r) = \left(1 - \frac{A^*(r)}{r}\right)^{-1}, \qquad (4)$$

$$\frac{dA(r)}{dr} = 8\pi\rho r^2 , \qquad (5)$$

and

$$\frac{dP(r)}{dr} = -\frac{(\rho + P)}{2}\frac{A'(r)}{A(r)} = -\frac{(\rho + P)}{2}\frac{8\pi P r^3 + A^*(r)}{r(r - A^*(r))} \qquad (6)$$

The last equation is known as the Tolman-Oppenheimer-Volkov (TOV) equation (Tolman, 1939; Oppenheimer & Volkov, 1939). The accumulated mass of the star, $M(r)$, is given by $A^*(r) = 2M(r)$. It has been shown that the chemical potential defined through the background metric in the form of $\mu = \varepsilon_F / \sqrt{A(r)}$ will automatically solve the TOV equation (Boer et al., 2009). So

$$\frac{\mu'(r)}{\mu(r)} = -\frac{1}{2}\frac{A'(r)}{A(r)}, \qquad (7)$$

and the TOV equation becomes

$$\frac{dP(r)}{dr} = (\rho + P)\frac{\mu'(r)}{\mu(r)} . \qquad (8)$$

From the first law of thermodynamics $\rho + P = \mu d$, the TOV takes the form

$$d\mu = \frac{1}{d}\left(\frac{\partial P}{\partial d}\right) d(d) . \qquad (9)$$

Obviously, the chemical potential can be determined, as a function of the number density:

$$\mu(d) = \int_0^d \frac{1}{\eta}\left(\frac{\partial P}{\partial \eta}\right) d\eta + \mu_{onset} \qquad (10)$$

Additionally, considering from the TOV equation together with the first law of thermodynamics, the density $d\rho = \mu d(d)$ can be integrated to

$$\rho(d) = \int_0^d \left[\int_0^\eta \frac{1}{\eta'}\left(\frac{\partial P}{\partial \eta'}\right) d\eta' + \mu_{onset}\right] d\eta . \qquad (11)$$

For a power-law equation of state, $P = kd^{\lambda}$, the chemical potential takes the following form,

$$\mu(d) = \frac{\lambda k}{\lambda - 1} d^{\lambda - 1} + \mu_{onset} , \qquad (12)$$

and eventually the equation of state is given by

$$\rho = \frac{1}{\lambda - 1} P + \mu_{onset} \left(\frac{P}{k} \right)^{1/\lambda}. \qquad (13)$$

In our holographic model of multiquarks, the relation between pressure and density has a unique power-law behavior, as also found by ? for the case of normal baryon (n_s =0). This is shown in Fig. 2. For small d, $P \propto d^2$ (n_s = 0) and for large d, $P \propto d^{7/5}$. The dependence on n_s becomes more significant when the density d is small that the equation of state becomes $P = \alpha_0 d^2 + \beta_0(n_s) d^4$.

RESULTS AND DISCUSSIONS

We can solve the TOV equation when the equations of state are given above by starting from the core of the star out to the surface. As we go from the center towards the surface of the star, the density decreases until it reaches a critical value ρ_c. This density corresponds to the number density dc where the power-law changes from $P \sim d^{7/5}$ to $P \sim d^2$ (see Fig. 2). For the crust region where the density $\rho < \rho_c$, multiquarks obey a different equation of state given by Eqn.(13). The radius of the core is defined to be the distance R_{core} where $\rho(R_{core}) = \rho_c$ and the surface of the star is defined to be the radial distance R where $\rho(R) = 0$.

For n_s = 0, numerical fittings suggest that $k = 10^{-0.4}$, $\lambda = 7/5$, d_c = 0.215443, μ_c = 0.564374 (*core*) and k' = 1; ,λ' = 2; μ_{onset} = 0.17495 (*crust*). For n_s = 0.3, good fit parameters are $k = 10^{-0.4}$, λ= 7/5, d_c = 0.086666, μ_c = 0.490069 (*core*) and a, b = 0.375, 180.0 ; $\lambda_{1,2}$ = 2, 4; μ_{onset} = 0.32767 (*crust*). Varying the central density ρ_0 of the star, we obtain the mass-density relation in Fig. 3.

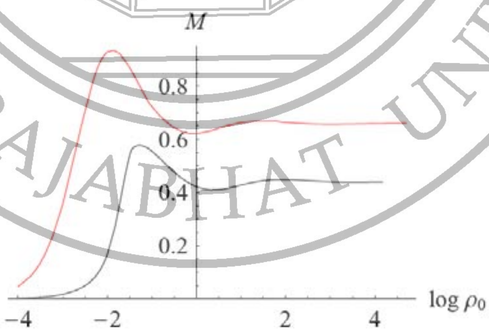

FIGURE 3. The relation between mass and central density of the multiquark star for n_s = 0 (upper), 0.3 (lower) (Burikham et al., 2010) .

Each curve has two maxima, a larger one in the small density region and a smaller one in the large density region. Interestingly, the contribution to the total mass of the multiquark star comes dominantly from the crust. This is shown in Fig.4. Even though the density is much lower, the volume of the crust is proportional to the second power of the radius and thus makes the contribution of the crust to the total mass larger than the core's.

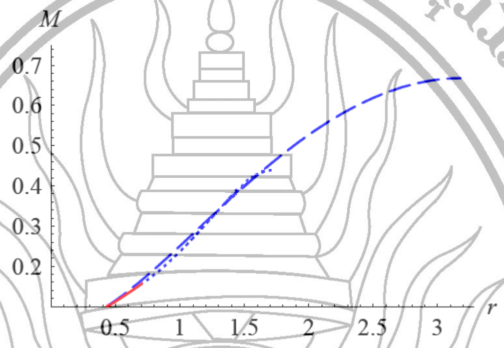

FIGURE 4. Comparison of the accumulated mass distribution in the hypothetical multiquark star for the central density $\rho_0 = 20$ between $n_s = 0$ and $n_s = 0.3$. The solid line represents the *core* while the dashed (dotted) line represent the *crust* for $n_s = 0$ (0.3) region (Burikham et al., 2010).

The pressure and density distribution within the multiquark star for the case of $n_s = 0(0.3)$ for the central density $\rho_0 = 20$ are shown in the Fig.5. It turns out that when the density and pressure reach the critical values where the equation of state changes into the different power-law for small *d*, the crust region continues for a large fraction of the total radius of the star making contribution of the crust mass to the total mass of the star dominant as shown in Fig. 4.

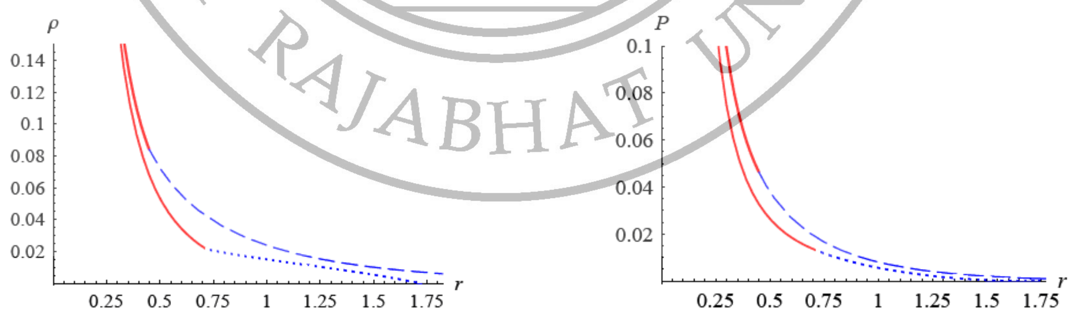

FIGURE 5. Comparison of the density, and pressure distribution in the hypothetical multiquark star for the central density ρ_0= 20 between n_s = 0 and 0.3. The (dashed)

dotted line represents the crust region of multiquark star with n_s = 0.3 (0). The solid lines represent the core region of which both cases are almost the same (Burikham et al., 2010).

The multiquark star with n_s = 0.3 (having color charges) converge to a smaller mass and radius at high central density (Fig.7). Multiquarks with color charges has lower pressure (and therefore smaller density) than the colorless ones for small density (Fig.5). In more realistic situations, all of the possible multiquarks with varying n_s coexist in the multiquark phase. The mass limit and mass radius relation will vary between the two typical cases we consider here. Since the equations of state are found NOT to be sensitive to the temperature, our results should also be valid even when the temperature varies within the star within the range between the gluon deconfinement and the chiral symmetry restoration.

Some remarks should be made regarding the hydro- dynamic properties of the multiquark phase (taken as nuclear liquid). At constant temperature and entropy, we can define the adiabatic index

$$\Gamma \equiv \frac{\rho}{P}\frac{\partial P}{\partial \rho} \quad (14)$$

$$= \frac{\rho}{P}c_s^2 \quad (15)$$

where c_s is the sound speed in the multiquark liquid. They depend on the equation of state of the multiquark and their distributions within the multiquark star are shown in Fig.6 for n_s = 0. The sound speed never exceeds the speed of light in vacuum. It is also found that the adiabatic index and the sound speed change within a small fraction as the central densities are varied for a given n_s.

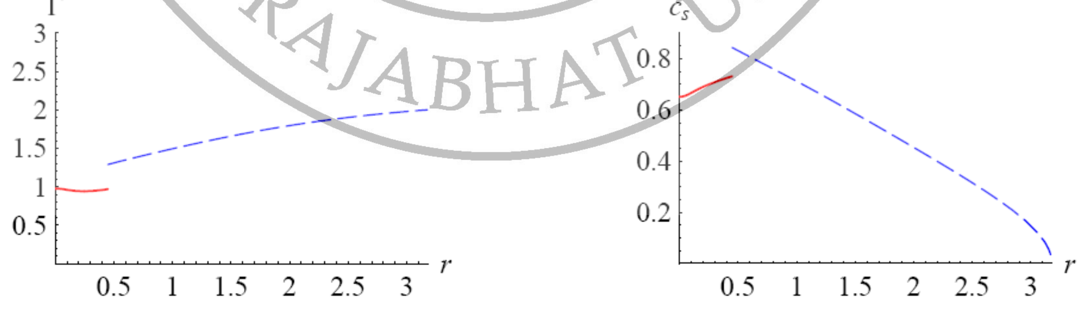

FIGURE 6. The adiabatic index at constant entropy (Γ) and the sound speed (c_s) distribution in the hypothetical multiquark star for the central density ρ_0 = 20 and n_s = 0. The inner (outer) solid (dashed) line represents the core (crust) region (Burikham et al., 2010).

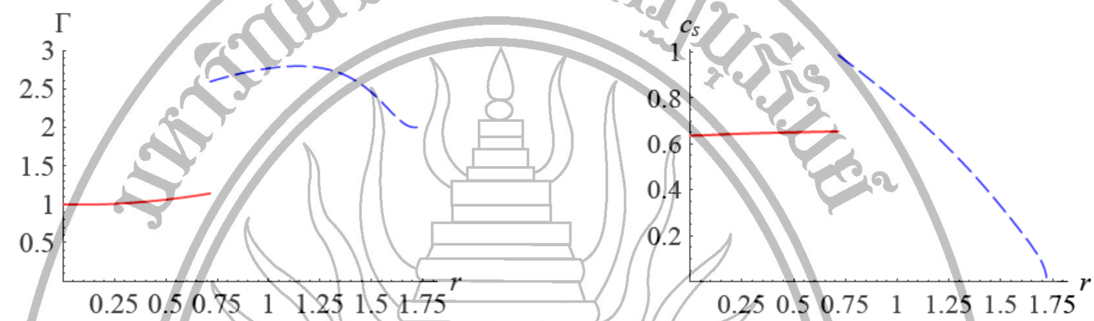

FIGURE 7. The adiabatic index at constant entropy (□) and the sound speed (c_s) distribution in the hypothetical multiquark star for the central density $_0$ = 20 and n_s = 0.3. The inner (outer) solid (dashed) line represents the core (crust) region (Burikham et al., 2010).

The spiral relation between mass and radius of the multiquark star is shown in Fig. 8. As the central density is increasing, the mass and radius of the n_s = 0 (0.3) multiquark star converge to the value of 0.659 (0.440) and 3.132 (1.704) respectively. For the core, the mass and radius of the core for n_s = 0 (0.3) converge to the value of 0.108 (0.169) and 0.471 (0.737).

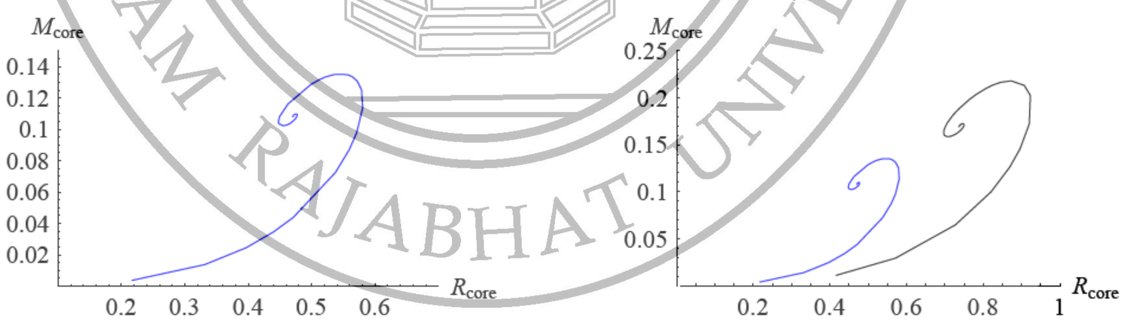

FIGURE 8. The relation between mass and radius of the core of the multiquark star with (a) n_s = 0, (b) n_s = 0 (left) and n_s = 0.3 (right) (Burikham et al., 2010).

We would like to estimate these limits of mass and radius in the physical units. Since our dimensionless quantities are related to the physical quantities through conversion factors given in Table 1 (Appendix A), both physical mass and radius vary with the energy density of the nuclear matter phase as $1/\sqrt{\textit{energy density scale}}$. For a multiquark nuclear phase with energy density scale 10 GeV/fm^3, the conversion factor of the mass and radius are 5.91$M_{\odot}$ and 8.71 km respectively. This would correspond to the converging mass and radius (in the limit of very large central density) of 3.89 (2.60) $M_{\odot}$ and 27.29 (14.85) km for n_s = 0 (0.3) multiquark star respectively.

In realistic situation, the nuclear phase in the outer region could lose heat out to the space in the form of radiation. The nuclear matter in the outer region of the crust will cool down and mostly become confined into neutrons and hadrons (e.g. hyperons, pions). Consequently the crust radius will become shorter than the estimated one and the multiquark star to be smaller and less massive than that of hypothetical prototype. For example, for the energy density scale 10 GeV/fm^3, the critical density is $\rho_c \sim 1.5 \times 10^{18}$ kg/m^3 ($n_s = 0$). This is still a sufficiently large density for the neutron layer to be formed. If the temperature of the nuclear matter in the crust region falls below the deconfinement temperature, the multiquarks will be confined into extremely dense neutrons and hadrons instead. For a typical neutron star, the distance of the neutron layer out to the star surface is roughly 5-6 km (Weber, 2005). If we add this number to the radius of the multiquark core, 0.471 × 8.71 ~ 4.10 km, we end up with a more realistic estimation for the multiquark star with radius ~ 10 km. Regardless of the name, only the core region is in the deconfined multiquark phase and the content of the outer layers are the confined nucleons.

CONCLUSION

Using the power-law equations of state for both small and large density regions, a spherically symmetric Einstein field equation is solved to obtain the Tolman-Oppenheimer-Volkov equation. By solving the equation numerically, it turns out that the multiquark star is separated into two layers, a core with higher density and a crust with lower density. Mass limit curve is also obtained as well as the mass sequence plot between the mass and radius of the multiquark star. They show typical spiral behavior of the star sequence plots. The mass limit curve shows two peaks corresponding to the equation of state of the small and the large density. Analyses show that the most contribution to the total mass is mainly from the crust. The adiabatic index at constant entropy, Γ, and the sound speed, c_s, of the multiquark nuclear phase within the star are calculated numerically. For large density, Γ is approximately close to 1 and c_s is roughly within range 0.6 - 0.7 of the speed of light. For small density, Γ is in the range 1.3 - 2.0 (2.0 - 3.0) and c_s is roughly 0 - 0.85 (0 - 0.99) for multiquark with $n_s = 0$ (0.3).

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APPENDIX

A. Dimensional translation table (Burikham, et al.,2010)

quantity	Dimesionless variable	Physical variable
pressure	P	$\left(c^4/Gr_0^2\right)P$
density	ρ	$\left(c^2/Gr_0^2\right)\rho$
mass	M	$\left(r_0c^2/G\right)M$
radius	r	$r_0 r$

where $r_0 \equiv \left(\dfrac{GN}{c^4\tau V_3}\right)^{-1/2} = \left(\dfrac{G}{c^4}(energy\ density\ scale)\right)^{-1/2}$

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