สนามอีเทอร์ในกาลอวกาศแบบ $M^{1+3} \times T^2$

AETHER COMPACTIFICATION IN $M^{1+3} \times T^2$ SPACETIME

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ในบทความนี้เราได้ทำการศึกษาสนามอีเทอร์ซึ่งเป็นสนามเวกเตอร์ที่ละเมิดสมมาตรลอเร็นต์ซในกาลอวกาศแบบ $M^{1+3} \times T^2$ โดยกำหนดให้ค่าเจาะจงของสนามไม่เป็นศูนย์ในทิศทางของมิติเพิ่มเติม แม้ว่าจากการคำนวณเราพบว่าเทนเซอร์พลังงาน-โมเมนตัมของสนามอีเทอร์ไม่เท่ากับศูนย์ก็ตามหลังจากกำหนดเงื่อนไขให้มิติเพิ่มเติมมีเสถียรภาพแต่ก็มีค่าลดลงแบบเอ็กโพเนนเชียล ดังนั้นจึงมีความเป็นไปได้ว่ามิติเพิ่มเติมอาจจะมีเสถียรภาพในแบบจำลองนี้ ต่อจากนั้นจะพิจารณาอันตกิริยาระหว่างสนามอีเทอร์กับสนามสเกลาร์ซึ่งพบว่าพลังงานสถานะกระตุ้นของสถานะคาลูซา-ไคลน์จะขึ้นอยู่กับรูปร่างของมิติเพิ่มเติมและทิศทางของสนามอีเทอร์

คำสำคัญ : มิติเพิ่มเติม, การละเมิดสมมาตรลอเร็นต์ซ, พลังงานมืด

ABSTRACT

We consider a space-like aether field with a fixed expectation value along the extra dimensions in an $M^{1+3} \times T^2$ spacetime. The crucial property in our model is the energy-momentum tensor associated with the aether has not vanished and it decays with an exponential rate. These imply that the aether field would inevitably contribute to the potential and possibly stabilize the extra dimensions. In addition, the interactions of aether with scalar field in our background lead to a modified dispersion relation that increases the mass of the KK excitation. This mass depends on both the moduli of the extra dimensions and the angle of aether field in its own parameter space.

Keywords: extra dimensions, Lorentz-violation, dark energy

Introduction

Data from Type Ia Supernovae Observation and Cosmic Microwave Background (CMB) suggested that the universe consists of the unknown sort of energy, namely dark energy which is responsible for accelerated expansion of our 4-dimensional spacetime. Unfortunately, the origin of dark energy is still not well understood for physicists. Recently, it was found by Greene and Levin that the Casimir energy of certain combinations of massless and massive fields in space with extra dimensions could play a crucial role of dark energy and it also stabilizes the extra dimensions [4]. However, if there are non-relativistic matter fields in the universe then this model cannot stabilize the extra dimensions. Chatrabhuti, Patcharamaneepakorn and Wongjun proposed a way to solve this problem by introducing the aether field into their model, the result showed that it can slow down the acceleration of the extra dimension and let the fine-tuned Casimir energy stabilizes the extra dimension successfully [2,3]. However, to solve the hierarchy problem, a six dimensional spacetime is preferred [1].

In this paper we will investigate the model in $M^{1+3}\times T^2$ spacetime by studying the Lorentz-violating field, called aether with non vanishing expectation value aligned along the direction of extra dimensions. The interactions between aether and scalar field have also been discussed.

Aether field in $M^{1+3}\times T^2$ spacetime

We consider on a product space $M^{1+3}\times T^2$, namely a product space between a 4-dimensional spacetime and 2-dimensional toroidally-compactified space. Let's define the coordinates $x^a=\{x^\mu, x^5, x^6\}$ where x^μ, with μ = 0,…,3, are non-compact coordinates and $x^5, x^6\in[0,2\pi]$ compact coordinates. We assume the cosmological ansatz

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + h_{ij}(x)dx^i dx^j\,, \tag{1}$$

where the metric h_{ij} represents the two dimensional compact space with i, j=5, 6 and takes the form

$$(h_{ij}) = \frac{b^2(x)}{\tau_2(x)} \begin{pmatrix} 1 & \tau_1(x) \\ \tau_1(x) & |\tau(x)|^2 \end{pmatrix} .$$

(2)

Note that $\tau = \tau_1 + i\tau_2$ is the shape moduli and the radion field $b(x)$ parameterizes the size of the extra dimensions.

Now we consider the aether field u^a is of spacelike-vector type, and we can define a field strength tensor as

$$V_{ab} = \nabla_a u_b - \nabla_b u_a \quad . \tag{3}$$

Even though the aether field is not related to the electromagnetic potential A_a, its field strength tensor is in the Maxwell form. Demand that the norm of the aether field is fixed, we write the action with the constraint $u_a u^a = v^2$ as

$$S = \int d^6 x \sqrt{-gh} \left[-\frac{1}{4} V_{ab} V^{ab} - \lambda (u_a u^a - v^2) + \sum_i L_i \right]. \tag{4}$$

Note that L_i represent various interaction terms between aether field and other fields such as boson which we will consider later.

By varying the action with respect to the aether field, we obtain the equation of motion for u^a, neglecting the interaction terms,

$$\nabla_a V^{ab} + v^{-2} u^b u_c \nabla_d V^{cd} = 0 . \tag{5}$$

To find the background solution we impose the aether field points along the extra dimensions, so that

$$u^a = (0,0,0,0,u^5,u^6) ,$$

(6)

this solution must satisfy the constraint equation, that is

$$v^2 = \frac{b^2}{\tau_2}(u^5)^2 + \frac{2b^2\tau_1}{\tau_2} u^5 u^6 + \frac{b^2|\tau|^2}{\tau_2}(u^6)^2 .$$

(7)

Using the complete square method we parameterize the solution in our background as

$$u^a = \left(0,0,0,0, \frac{v\sqrt{\tau_2}}{b}\left(\cos\theta - \frac{\tau_1 \sin\theta}{\tau_2} \right), \frac{v\sin\theta}{b\sqrt{\tau_2}} \right),$$

(8) where $\theta = \theta(t)$ is an angle parameter that depends only on time. However, this configuration needs to satisfy the equation of motion. Substitute into Eq.(5), we obtain the equation describing dynamics of the parameter $\theta(t)$

$$
\begin{aligned}
0 = & \frac{1}{8b\tau_2^{9/2}} v(\tau_1 \cos\theta + \tau_2 \sin\theta)(-8H_b\tau_2^4\dot{\theta} - 2H_b\tau_2\dot{\tau}_1 + 2H_b\tau_2\dot{\tau}_1\cos 2\theta - 4H_b\tau_1^2\tau_2\dot{\tau}_1 + 4H_b\tau_1^2\tau_2\dot{\tau}_1\cos 2\theta \\
& -2H_b\tau_1^4\tau_2\dot{\tau}_1 + 2H_b\tau_1^4\tau_2\dot{\tau}_1\cos 2\theta + 4H_b\tau_1\tau_2^2\dot{\tau}_1\sin 2\theta + 4H_b\tau_1^3\tau_2^2\dot{\tau}_1\sin 2\theta - 12H_a\tau_2^3\dot{\tau}_1\cos 2\theta - 4H_b\tau_2^3\dot{\tau}_1 \\
& +12H_b\tau_2^3\dot{\tau}_1\cos 2\theta - 4H_b\tau_1^2\tau_2^3\dot{\tau}_1\cos 2\theta - 4H_b\tau_1\tau_2^4\dot{\tau}_1\sin 2\theta + 2H_b\tau_2^5\dot{\tau}_1\cos 2\theta + 4\tau_2^3\dot{\tau}_1\dot{\theta}\sin 2\theta \\
& -\dot{\tau}_1^2\sin 2\theta - 2\tau_1^2\dot{\tau}_1^2\sin 2\theta - \tau_1^4\dot{\tau}_1^2\sin 2\theta + 2\tau_1\tau_2\dot{\tau}_1^2 + 2\tau_1^3\tau_2\dot{\tau}_1^2 - 2\tau_2^2\dot{\tau}_1^2\sin 2\theta - 2\tau_1\tau_2^3\dot{\tau}_1^2 \\
& +\tau_2^4\dot{\tau}_1^2\sin 2\theta + 12a^2H_a\tau_2^3(2\tau_2\dot{\theta} + \dot{\tau}_1 - 4H_b\tau_1\tau_2^2\dot{\tau}_2 + 4H_b\tau_1\tau_2^2\dot{\tau}_2\cos 2\theta - 4H_b\tau_1^3\tau_2^2\dot{\tau}_2 + 4H_b\tau_1^3\tau_2^2\dot{\tau}_2 \\
& +4H_b\tau_1^3\tau_2^2\dot{\tau}_2\cos 2\theta - 12H_a\tau_2^3\dot{\tau}_2\sin 2\theta + 16H_b\tau_2^3\dot{\tau}_2\sin 2\theta + 8H_b\tau_1^2\tau_2^3\dot{\tau}_2\sin 2\theta - 4H_b\tau_1\tau_2^4\dot{\tau}_2 \\
& -4H_b\tau_1\tau_2^4\dot{\tau}_2\cos 2\theta - 4\tau_2^3\dot{\tau}_2\dot{\theta}\cos 2\theta - \dot{\tau}_1\dot{\tau}_2 + \dot{\tau}_1\dot{\tau}_2\cos 2\theta - 2\tau_1^2\dot{\tau}_1\dot{\tau}_2 + 2\tau_1^2\dot{\tau}_1\dot{\tau}_2\cos 2\theta - \tau_1^4\dot{\tau}_1\dot{\tau}_2 \\
& +\tau_1^4\dot{\tau}_1\dot{\tau}_2\cos 2\theta - 2\tau_1\tau_2\dot{\tau}_1\dot{\tau}_2\sin 2\theta - 2\tau_1^3\tau_2\dot{\tau}_1\dot{\tau}_2\sin 2\theta + 6\tau_2^2\dot{\tau}_1\dot{\tau}_2 + 6\tau_2^2\dot{\tau}_1\dot{\tau}_2\cos 2\theta + 6\tau_1^2\tau_2^2\dot{\tau}_1\dot{\tau}_2 \\
& -2\tau_1\tau_2^3\dot{\tau}_1\dot{\tau}_2\sin 2\theta - \tau_2^4\dot{\tau}_1\dot{\tau}_2 - \tau_2^4\dot{\tau}_1\dot{\tau}_2\cos 2\theta - 2\tau_1\tau_2\dot{\tau}_2^2 + 2\tau_1\tau_2\dot{\tau}_2^2\cos 2\theta - 2\tau_1^3\tau_2\dot{\tau}_2^2 + 2\tau_1^3\tau_2\dot{\tau}_2^2\cos 2\theta \\
& +4\tau_2^2\dot{\tau}_2^2\sin 2\theta + 2\tau_1\tau_2^3\dot{\tau}_2^2 + 2\tau_1\tau_2^3\dot{\tau}_2^2\cos 2\theta - b^2\tau_2(2\tau_2\dot{\theta} + \dot{\tau}_1)(-4H_b\tau_2(\cos^2\theta + \tau_1^2\cos^2\theta + \tau_1\tau_2\sin 2\theta \\
& +\tau_2^2\sin^2\theta) - (\tau_1^2\sin 2\theta + 2\tau_1\tau_2 + \sin 2\theta(1+\tau_2^2)\dot{\tau}_1 + 2(\cos^2\theta + \tau_1^2\cos^2\theta - \tau_2^2\sin^2\theta)\dot{\tau}_2) - 8\tau_2^4\ddot{\theta} \\
& -4\tau_2^3\ddot{\tau}_1 - 4\tau_2^3\ddot{\tau}_1\cos 2\theta - 4\tau_2^3\ddot{\tau}_2)\sin 2\theta
\end{aligned} \tag{9}
$$

where $H_a = \dfrac{\dot{a}}{a}$, $H_b = \dfrac{\dot{b}}{b}$.

Energy- momentum tensor of the aether field

By varying the action with respect to the metric, we obtain the energy-momentum tensor from the aether field in the following form,

$$T_{ab} = V_{ac}V_b{}^c - \frac{1}{4}V_{cd}V^{cd}g_{ab} + v^{-2}u_a u_b u_c \nabla_d V^{dc}. \tag{10}$$

Raising an index to obtain:

$$
\begin{aligned}
T^0{}_0 = & -\frac{1}{8\tau_2^2} 3v^2(4H_b\tau_2^2 + 4\tau_2^2\dot{\theta}^2 + 4\dot{\tau}_1^2\cos^2\theta + 2\dot{\tau}_1\dot{\tau}_2\sin 2\theta + \dot{\tau}_2^2 \\
& +4H_b\tau_2(\dot{\tau}_1\sin 2\theta - \dot{\tau}_2\cos 2\theta) + 8\tau_2\dot{\theta}\cos\theta(\dot{\tau}_1\cos\theta + \sin\theta\dot{\tau}_2))
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
T^1{}_1 = & \frac{1}{8\tau_2^2} v^2(4H_b^2\tau_2^2 + 4\tau_2^2\dot{\theta}^2 + 4\dot{\tau}_1^2\cos^2\theta + 2\dot{\tau}_1\dot{\tau}_2\sin 2\theta + \dot{\tau}_2^2 \\
& +4H_b\tau_2(\dot{\tau}_1\sin 2\theta - \dot{\tau}_2\cos 2\theta) + 8\tau_2\dot{\theta}\cos\theta(\dot{\tau}_1\cos\theta + \dot{\tau}_2\sin\theta))
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&T^{5}{}_{5}=-\frac{1}{32\tau_2^5}v^2(-16H_b^2\tau_2^4(-\tau_1\sin 2\theta+(2+\cos 2\theta)\tau_2)+4b^2H_b\tau_2^2\cos\theta(4\tau_1^3\cos\theta\sin^2\theta+\\
&(\sin\theta-3\sin 3\theta)\tau_1^2\tau_2+4\tau_2\cos^2\theta\sin\theta(-1+\tau_2^2)+\tau_1(4\cos\theta\sin^2\theta+(\cos\theta+3\tau_2^2\cos 3\theta)\tau_2^2))\\
&(2\tau_2\dot{\theta}+\dot{\tau}_1)+48H_a\tau_2^3\cos\theta(-\tau_1\sin\theta+\tau_2\cos\theta)(\dot{\tau}_1\sin 2\theta-\dot{\tau}_2\cos 2\theta)+8\tau_1^5\dot{\tau}_1\cos^2\theta\sin\theta\\
&(\dot{\tau}_1\cos\theta+\dot{\tau}_2\sin\theta+4b^2\tau_2\cos\theta(-\tau_1\sin\theta+\tau_2\cos\theta)(2\tau_2\dot{\theta}+\dot{\tau}_1)(2(\cos^2\theta+\tau_1^2\cos^2\theta-\\
&\tau_2^2\sin^2\theta)\dot{\tau}_1+(\tau_1^2\sin 2\theta+2\tau_1\tau_2+\sin 2\theta(1+\tau_2^2))\dot{\tau}_2+\tau_1^4\tau_2(-8\dot{\tau}_1^2\cos^4\theta+(6\sin 2\theta+\sin 4\theta)\dot{\tau}_1\dot{\tau}_2\\
&+4\dot{\tau}_2^2\sin^2 2\theta)+2\tau_1^3(2\sin 2\theta(1+\cos 2\theta-\tau_2^2)\dot{\tau}_1^2-4\cos^2\theta(-2\sin^2\theta+(2+\cos 2\theta)\tau_2^2)\dot{\tau}_1\dot{\tau}_2+\\
&8\tau_2^2\dot{\tau}_2^2\cos\theta\sin^3\theta)+2\tau_1^2\tau_2(-4\cos^2\theta(1+\cos 2\theta-\tau_2^2)\dot{\tau}_1^2+2\sin 2\theta(1+(-2+\cos 2\theta)\tau_2^2)\dot{\tau}_1\dot{\tau}_2\\
&+2(\sin^2 2\theta-4\tau_2^2\cos^4\theta)\dot{\tau}_2^2)-2\tau_2(-2H_b\tau_1^5\dot{\tau}_1\sin^2 2\theta+2H_b\tau_1^4\tau_2\sin 2\theta((1+3\cos 2\theta)\dot{\tau}_1-\\
&2\dot{\tau}_2\sin 2\theta-4H_b\tau_1^3\cos\theta(2(\sin\theta\sin 2\theta+\tau_2^2\cos 3\theta)\dot{\tau}_1+(\sin\theta-3\sin 3\theta)\tau_2^2\dot{\tau}_2)-4H_b\tau_1^2\tau_2\cos\theta\\
&(2\sin 3\theta(-1+\tau_2^2)\dot{\tau}_1+(4\cos\theta\sin^2\theta+(\cos\theta+3\tau_2^2\cos 3\theta)\tau_2^2)\dot{\tau}_2)+2\tau_2(-8\tau_2^3\cos\theta(3H_aH_b\cos\theta\\
&+H_b^2\cos\theta+\dot{H}_b\cos\theta-2H_b\dot{\theta}\sin\theta)+4H_b\dot{\tau}_1\cos^3\theta\sin\theta+4H_b\tau_2^4\dot{\tau}_1\cos^3\theta+H_b\tau_2^2((10\sin 2\theta+\\
&3\sin 4\theta)\dot{\tau}_1-4(2\cos 2\theta+\cos 4\theta)\dot{\tau}_2))+\tau_1(8\tau_2^3(3H_aH_b\sin 2\theta+H_b^2\sin 2\theta+\dot{H}_b\sin 2\theta+\\
&2H_b\dot{\theta}\cos 2\theta)+(-1+\cos 4\theta)H_b\dot{\tau}_1+4H_b\tau_2^4\cos^2\theta((-1+3\cos 2\theta)\dot{\tau}_1-2\dot{\tau}_2\sin 2\theta+4H_b\tau_2^2\cos\theta\\
&(2\dot{\tau}_1\cos 3\theta+(-3\sin\theta+5\sin 3\theta)\dot{\tau}_2)))-\tau_2^2(4\cos^2\theta(5+\cos 2\theta)\dot{\tau}_1^2+(10\sin 2\theta+3\sin 4\theta)\dot{\tau}_1\dot{\tau}_2\\
&-2(-2+\cos 4\theta)\dot{\tau}_2^2)+4\tau_2^4((2+4\cos 2\theta)\dot{\theta}^2+\cos^2\theta\sin\theta\dot{\tau}_1(\sin\theta\dot{\tau}_1-\cos\theta\dot{\tau}_2))-8\cos^2\theta\tau_2^3\\
&(\dot{\theta}((-4+\cos 2\theta)\dot{\tau}_1+\sin 2\theta\dot{\tau}_2)+\sin 2\theta\ddot{\tau}_1-\cos 2\theta\ddot{\tau}_2))+\tau_1(8\dot{\tau}_1\cos^2\theta\sin\theta(\dot{\tau}_1\cos\theta+\dot{\tau}_2\sin\theta)\\
&-2\tau_2^4\cos\theta(-32\dot{\theta}^2\sin\theta-4\dot{\tau}_1^2\sin^3\theta+(-5\cos\theta+\cos 3\theta)\dot{\tau}_1\dot{\tau}_2+8\dot{\tau}_2^2\cos^2\theta\sin\theta)+2\tau_2^2\\
&((6\sin 2\theta+\dot{\tau}_1^2\sin 4\theta-4\cos^2\theta(-3+4\cos 2\theta)\dot{\tau}_1\dot{\tau}_2+24\dot{\tau}_2^2\cos\theta\sin^3\theta)+4\tau_2^3(\dot{\theta}(-(-8\sin 2\theta\\
&+\sin 4\theta)\dot{\tau}_1+(3+\cos 4\theta)\dot{\tau}_2)+(-1+\cos 4\theta)\ddot{\tau}_1+\ddot{\tau}_2\sin 4\theta)))
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&T^{6}{}_{6}=\frac{1}{16\tau_2^5}v^2(-8H_b\tau_2^4(-\tau_1\sin 2\theta+(-2+\cos 2\theta)\tau_2)+2b^2H_b\tau_2^2\sin\theta(4\tau_1^3\cos^2\theta\sin\theta-\\
&(\cos\theta+3\cos 3\theta)\tau_1^2\tau_2-4\tau_2\cos\theta\sin^2\theta(-1+\tau_2^2)+\tau_1(\sin\theta+\sin 3\theta+(\sin\theta-\\
&3\sin 3\theta)\tau_2^2))(2\tau_2\dot{\theta}+\dot{\tau}_1)-24\tau_2^3H_a\sin\theta(\tau_1\cos\theta+\tau_2\sin\theta)(\dot{\tau}_1\sin 2\theta-\cos 2\theta\dot{\tau}_2)+\\
&4\tau_1^5\dot{\tau}_1\cos^2\theta\sin\theta(\dot{\tau}_1\cos\theta+\dot{\tau}_2\sin\theta)-2b^2\tau_2\sin\theta(\tau_1\cos\theta+\tau_2\sin\theta)(2\tau_2\dot{\theta}+\dot{\tau}_1)(2(\cos^2\theta\\
&+\tau_1^2\cos^2\theta-\tau_2^2\sin^2\theta)\dot{\tau}_1+(\tau_1^2\sin 2\theta+2\tau_1\tau_2+\sin 2\theta(1+\tau_2^2))\dot{\tau}_2)+\tau_1^4\tau_2(\sin 2\theta\dot{\tau}_1^2+\\
&(5+\cos 2\theta)\dot{\tau}_1\dot{\tau}_2+2\dot{\tau}_2^2)+2\tau_1^3\sin\theta(2\cos\theta(1+\cos 2\theta-\tau_2^2)\dot{\tau}_1^2+2\sin\theta(2\cos^2\theta+\\
&(2+\tau_2^2\cos 2\theta)\dot{\tau}_1\dot{\tau}_2-(-5\cos\theta+\cos 3\theta)\tau_2^2\dot{\tau}_2^2)+4\tau_1^2\tau_2\sin\theta(\sin\theta(1+\cos 2\theta-\tau_2^2)\dot{\tau}_1^2+\\
&\cos\theta(3+(-2+\cos 2\theta)\tau_2^2)\dot{\tau}_1\dot{\tau}_2+\sin\theta(2\cos^2\theta+(3+\cos 2\theta)\tau_2^2)\dot{\tau}_2^2)+\tau_2(2H_b\tau_1^5\dot{\tau}_1\sin^2 2\theta\\
&+2H_b\tau_1^4\tau_2\sin 2\theta((1-3\cos 2\theta)\dot{\tau}_1+2\dot{\tau}_2\sin 2\theta)+4H_b\tau_1^3\sin\theta((\sin\theta+\sin 3\theta-2\tau_2^2\sin 3\theta)\dot{\tau}_1\\
&-(\cos\theta+3\cos 3\theta)\tau_2^2\dot{\tau}_2)+4H_b\tau_1^2\tau_2\sin\theta(2\cos 3\theta(-1+\tau_2^2)\dot{\tau}_1+(\sin\theta+\sin 3\theta+(\sin\theta-\\
&3\sin 3\theta)\tau_2^2)\dot{\tau}_2+2\tau_2(-8\sin\theta\tau_2^3(3H_a\dot{H}_b\sin\theta+H_b^2\sin\theta+\dot{H}_b\sin\theta+2H_b\dot{\theta}\cos\theta)+\\
&4\cos\theta H_b\sin^3\theta\dot{\tau}_1+4H_b\dot{\tau}_1\tau_2^4\cos\theta\sin^3\theta+H_b\tau_2^2((2\sin 2\theta-3\sin 4\theta)\dot{\tau}_1+4(-2\cos 2\theta\\
&+\cos 4\theta)\dot{\tau}_2))-\tau_1(8\tau_2^3(3H_aH_b\sin 2\theta+H_b^2\sin 2\theta+\dot{H}_b\sin 2\theta+2H_b\dot{\theta}\cos 2\theta)+\\
&(-1+\cos 4\theta)H_b\dot{\tau}_1-2\dot{\tau}_2\sin 2\theta)-4H_b\sin^2\theta\tau_2^4((1+3\cos 2\theta)\dot{\tau}_1-2\sin 2\theta\dot{\tau}_2+\\
&2H_b\tau_2^2(2(3\cos 2\theta+\cos 4\theta)\dot{\tau}_1+(-2\sin 2\theta+5\sin 4\theta)\dot{\tau}_2)))+\tau_1(4\dot{\tau}_1\cos^2\theta\sin\theta(\dot{\tau}_1\cos\theta\\
&+\dot{\tau}_2\sin\theta)+\tau_2^2((6\sin 2\theta+\sin 4\theta)\dot{\tau}_1^2+2(5+\cos 2\theta-2\cos 4\theta)\dot{\tau}_1\dot{\tau}_2+(10\sin 2\theta-\\
&3\sin 4\theta)\dot{\tau}_2^2)+2\tau_2^4(8\dot{\theta}^2\sin 2\theta+\sin^2\theta(\dot{\tau}_1^2\sin 2\theta+(-5+\cos 2\theta)\dot{\tau}_1\dot{\tau}_2+2\dot{\tau}_2^2\sin 2\theta))+2\tau_2^3\\
&(\dot{\theta}(-(-8\sin 2\theta+\sin 4\theta)\dot{\tau}_1+(3+\cos 4\theta)\dot{\tau}_2)+(-1+\cos 4\theta)\ddot{\tau}_1+\ddot{\tau}_2\sin 4\theta))-\\
&\tau_2(-4\dot{\tau}_1\cos\theta\sin^2\theta(\dot{\tau}_1\cos\theta+\dot{\tau}_2\sin\theta)+\tau_2^2((-1+8\cos 2\theta+\cos 4\theta)\dot{\tau}_1^2+(-2\sin 2\theta+\\
&3\sin 4\theta)\dot{\tau}_1\dot{\tau}_2-2(4-4\cos 2\theta+\cos 4\theta)\dot{\tau}_2^2)+4\tau_2^4((2+4\cos 2\theta)\dot{\theta}^2+\dot{\tau}_1\sin^3\theta(-\dot{\tau}_1\sin\theta+\\
&\dot{\tau}_2\cos\theta))+2\tau_2^3(\dot{\theta}(-(1-10\cos 2\theta+\cos 4\theta)\dot{\tau}_1+8\cos\theta\sin^3\theta\dot{\tau}_2+\\
&4\sin^2\theta(\ddot{\tau}_1\sin 2\theta-\ddot{\tau}_2\cos 2\theta))))
\end{aligned}
$$

(14)

$$
\begin{aligned}
T^{6}{}_{5} = {} & \frac{1}{32\tau_2^5} v^2(16H_b^2\tau_2^4 \sin 2\theta - 4b^2 H_b \tau_2^2 \sin 2\theta(-\tau_1^2 \sin 2\theta + 2\tau_1\tau_2 \cos 2\theta + \\
& \sin 2\theta(-1+\tau_2^2))(2\tau_2\dot\theta + \dot\tau_1) - 24H_a\tau_2^3 \sin 2\theta(\dot\tau_1 \sin 2\theta - \dot\tau_2 \cos 2\theta) + 8\cos^2\theta \\
& \sin\theta(1+\tau_1^2)^2 \dot\tau_1(\dot\tau_1\cos\theta + \dot\tau_2 \sin\theta) + 4\tau_1 \sin 2\theta(1+\tau_1^2)\tau_2\dot\tau_2((2+\cos 2\theta)\dot\tau_1 + \\
& \dot\tau_2 \sin 2\theta) - 2b^2\tau_2 \sin 2\theta(2\tau_2\dot\theta + \dot\tau_1)(2(\cos^2\theta + \tau_1^2\cos^2\theta - \tau_2^2 \sin^2\theta)\dot\tau_1 + \\
& (\tau_1^2 \sin 2\theta + 2\tau_1\tau_2 + \sin 2\theta(1+\tau_2^2))\dot\tau_2) + 4\tau_2^2\cos\theta(2\sin\theta(3+\cos 2\theta - \tau_1^2)\dot\tau_1^2 + \\
& (7\cos\theta - 3\cos 3\theta)\dot\tau_1\dot\tau_2 + 4\sin\theta(2-\cos 2\theta + \tau_1^2)\dot\tau_2^2) + 4\tau_2^4 \sin 2\theta(8\dot\theta^2 + \dot\tau_1 \sin\theta \\
& (\dot\tau_1\sin\theta - \dot\tau_2\cos\theta)) + 4\tau_2(H_b \sin^2 2\theta(1+\tau_1^2)^2\dot\tau_1 + H_b\tau_2^4\dot\tau_1 \sin^2 2\theta + 2H_b\tau_1 \sin 2\theta \\
& (1+\tau_1^2)\tau_2(-\dot\tau_1\cos 2\theta + \dot\tau_2 \sin 2\theta) - 2H_b\tau_2^2\cos\theta((\cos\theta + 3\cos 3\theta + 4\tau_1^2\cos\theta\sin^2\theta)\dot\tau_1 \\
& +2(-\sin\theta + \sin 3\theta)(2+\tau_1^2)\dot\tau_2) - \tau_2^3(12H_aH_b \sin 2\theta + 4\sin 2\theta(H_b^2 + \dot H_b) \\
& +H_b(8\dot\theta\cos 2\theta + \tau_1(-\dot\tau_1 \sin 4\theta + 2\dot\tau_2 \sin^2 2\theta)))) + 2\tau_2^3(2\dot\theta(-(-8\sin 2\theta + \sin 4\theta)\dot\tau_1 + \\
& (3+\cos 4\theta)\dot\tau_2) + 2\sin 2\theta(\tau_1\dot\tau_2((-2+\cos 2\theta)\dot\tau_1 + \dot\tau_2 \sin 2\theta) - 2\ddot\tau_1 \sin 2\theta + 2\ddot\tau_2 \cos 2\theta)))
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
T^{5}{}_{6} = {} & \frac{1}{4\tau_2^5} v^2(\tau_2^2(-\tau_1 \sin\theta + \tau_2\cos\theta)(\tau_1\cos\theta + \tau_2\sin\theta)(2\tau_2\dot\theta + \dot\tau_1)^2 - \\
& \tau_2^2(-\tau_1 \sin\theta + \tau_2\cos\theta)(2\tau_2\dot\theta + \dot\tau_1)(2H_b\tau_2(-\tau_1\sin\theta + \tau_2\cos\theta) + \tau_2(\dot\tau_1 \sin\theta \\
& -\dot\tau_2\cos\theta) - \tau_1(\dot\tau_1\cos\theta + \dot\tau_2\sin\theta)) + \tau_2^2(\tau_1\cos\theta + \tau_2\sin\theta)(2\tau_2\dot\theta + \dot\tau_1)(2H_b\tau_2 \\
& (\tau_1\cos\theta + \tau_2\sin\theta) + \tau_1(\dot\tau_1\sin\theta - \dot\tau_2\cos\theta) + \tau_2(\dot\tau_1\cos\theta + \dot\tau_2\sin\theta)) - \tau_2^2(2H_b\tau_2 \\
& (-\tau_1\sin\theta + \tau_2\cos\theta) + \tau_2(\dot\tau_1\sin\theta - \dot\tau_2\cos\theta) - \tau_1(\dot\tau_1\cos\theta + \dot\tau_2\sin\theta))(2H_b\tau_2 \\
& (\tau_1\cos\theta + \tau_2\sin\theta) + \tau_1(\dot\tau_1\sin\theta - \dot\tau_2\cos\theta) + \tau_2(\dot\tau_1\cos\theta + \dot\tau_2\sin\theta)) - \\
& \frac{1}{2}(\tau_1\sin\theta - \tau_2\cos\theta)(\tau_1\cos\theta + \tau_2\sin\theta)(16H_b^2\tau_2^4 - 2b^2H_b\tau_2^2(-\tau_1^2\sin 2\theta + \\
& 2\tau_1\tau_2\cos 2\theta + \sin 2\theta(-1+\tau_2^2))(2\tau_2\dot\theta + \dot\tau_1) + 12H_a\tau_2^3(-\dot\tau_1\sin 2\theta + \dot\tau_2\cos 2\theta) \\
& +2\cos\theta(1+\tau_1^2)^2\dot\tau_1(\dot\tau_1\cos\theta + \dot\tau_2\sin\theta) + 2\tau_1(1+\tau_1^2)\tau_2\dot\tau_2((2+\cos 2\theta)\dot\tau_1 + \\
& \dot\tau_2\sin 2\theta) - b^2\tau_2(2\tau_2\dot\theta + \dot\tau_1)(2(\cos^2\theta + \tau_1^2\cos^2\theta - \tau_2^2\sin^2\theta)\dot\tau_1 + (\tau_1^2\sin 2\theta + \\
& 2\tau_1\tau_2 + \sin 2\theta(1+\tau_2^2))\dot\tau_2) + 2\tau_2^2((3+\cos 2\theta - \tau_1^2)\dot\tau_1^2 + 3\dot\tau_1\dot\tau_2\sin 2\theta + (3 - \\
& 2\cos 2\theta + 2\tau_1^2)\dot\tau_2^2) + \tau_2^4(8\dot\theta^2 + 2\dot\tau_1\sin\theta(\dot\tau_1\sin\theta - \dot\tau_2\cos\theta)) + 2\tau_2(H_b\sin 2\theta(1+\tau_1^2)^2 \\
& \dot\tau_1 + H_b\tau_2^4\dot\tau_1\sin 2\theta - 2H_b\tau_1(1+\tau_1^2)\tau_2(\dot\tau_1\cos 2\theta - \dot\tau_2\sin 2\theta) - 2H_b\tau_2^2(\sin 2\theta(-3+\tau_1^2)\dot\tau_1 \\
& +2\cos 2\theta(2+\tau_1^2)\dot\tau_2) - 2\tau_2^3(6H_aH_b + 2H_b^2 + 2\dot H_b - H_b\tau_1(\dot\tau_1\cos 2\theta - \dot\tau_2\sin 2\theta))) + \\
& 2\tau_2^3(-2\dot\theta((-2+\cos 2\theta)\dot\tau_1 + \dot\tau_2\sin 2\theta) + \tau_1\dot\tau_2((-2+\cos 2\theta)\dot\tau_1 + \dot\tau_2\sin 2\theta) - 2\ddot\tau_1\sin 2\theta \\
& +2\ddot\tau_2\cos 2\theta)))
\end{aligned}
\tag{16}
$$

The crucial property of the aether field in our model is that its energy-momentum tensor depends on the background spacetime and the field parameter $\theta(t)$, therefore if the extra dimensions are stabilized with $\dot{b},\dot{\tau}_1,\dot{\tau}_2 = 0$, its energy-momentum tensor dose not vanish but remains:

$$T^0{}_0 = -\frac{1}{2}v^2\dot{\theta}^2$$

(17)

$$T^1{}_1 = T^2{}_2 = T^3{}_3 = \frac{1}{2}v^2\dot{\theta}^2 \quad (18)$$

$$T^5{}_5 = -\frac{v^2\dot{\theta}^2}{2\tau_2}(-2\tau_1\sin 2\theta + (1+2\cos 2\theta)\tau_2) \quad (19)$$

$$T^6{}_6 = \frac{v^2\dot{\theta}^2}{2\tau_2}(2\tau_1\sin 2\theta + (1-2\cos 2\theta)\tau_2) \quad (20)$$

$$T^5{}_6 = \frac{v^2\dot{\theta}^2}{\tau_2}(-\tau_1^2\sin 2\theta + 2\tau_1\tau_2\cos 2\theta + \tau_2^2\sin 2\theta)$$

(21)

$$T^6{}_5 = \frac{v^2\dot{\theta}^2\sin 2\theta}{\tau_2} \quad (22)$$

and the equation of motion, Eq.(9) reduces to the form:

$$\ddot{\theta} = -3a^2H_a\dot{\theta}. \quad (23)$$

After the extra dimensions are stabilized, if the Hubble parameter H_a is constant in time then the scale factor $a(t) \propto e^{H_a t}$. Therefore, the solution for Eq. (23) are

$$\frac{\ddot{\theta}(t)}{\dot{\theta}(t)} = -3a^2H_a = -3a\dot{a}, \quad (24)$$

and

$$\dot{\theta}(t) = \exp[-\frac{3}{2}a^2] \quad (25)$$

These important results show that the aether field in our background contributes to the energy density and it decays with an exponential rate. Therefore, it will contribute to the effective potential generically.

Interaction of the aether on scalar field

We now consider the effect of the aether coupled to real massive scalar field. The simplest Lagrangian is

$$L_{\phi} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2\mu_{\phi}^2}u^a u^b \partial_a\phi\partial_b\phi, \tag{26}$$

this imposes a Z_2 symmetry, $u^a \to -u^a$ because if we have not imposed it the lowest order coupling is $-\mu_{\phi}^{-1}u^a\partial_a\phi = -\mu_{\phi}^{-1}(\partial_a u^a)\phi$ by integration by parts, which vanishes in our background solution for u^a. The equation of motion for this Lagrangian is

$$\partial_a\partial^a\phi - m^2\phi = -\mu_{\phi}^{-2}\partial_a(u^a u^b \partial_b\phi). \tag{27}$$

Expanding the scalar field in Fourier modes, we obtain the modified dispersion relation,

$$k_{\mu}k^{\mu} = m^2 + \frac{|\tau|^2}{b^2\tau_2}n_1^2 - \frac{2\tau_1}{b^2\tau_2}n_1 n_2 + \frac{1}{b^2\tau_2}n_2^2 +$$

$$\alpha_{\phi}^2[(\frac{\tau_1^2}{b^2\tau_2} - \frac{\tau_1\sin 2\theta}{b^2} + \frac{\tau_2\cos^2\theta}{b^2} - \frac{\tau_1^2\cos^2\theta}{b^2\tau_2})n_1^2 + (\frac{\sin 2\theta}{b^2} - \frac{2\tau_1\sin^2\theta}{b^2\tau_2})n_1 n_2 + \frac{\sin^2\theta}{b^2\tau_2}n_2^2] \tag{28}$$

where the dimensionless parameter $\alpha_{\phi} = v/\mu_{\phi}$ is the ratio of the aether vev to the coupling μ_{ϕ}. Momentum of the scalar field along the compactified extra dimension will be quantized as $k_5 = n_1$ and $k_6 = n_2$ in our spacetime geometry. Eq. (28) suggests that the mass gap between the different states in the KK tower is enhanced by the interaction with the aether field and the distortion of the extra dimensions parametrized by b, τ_1, τ_2. The mass also depends crucially on the angle parameter θ.

Conclusions

We have shown that the dispersion relation of the scalar field is modified by the presence of the Lorentz-violating aether field and the shape moduli. In our spacetime geometry, aether field rotates in extra dimensions with parameter $\theta(t)$ but the solution shows that it has an exponential decay. This implies that aether should rotate with a decreasing rate and could finally be fixed in its own parameter space with constant $\theta(t)$.

In the future work we will consider the aether field and Casimir energy of certain combination of massless and massive fields in the same background spacetime. We hope that the aether field together with the Casimir energy should be able to stabilize the extra dimensions of our universe in the matter dominating era. Once the extra dimensions are stabilized, it is possible to interpret the Casimir energy as the dark energy which is driving the accelerated expansion of the universe in the current epoch.

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