Fibonacci Q-matrix and Matrices Formula for Fibonacci

and Lucas Sequences

Teerapan Jodnok¹ Sukanya Somprom²

¹Department of Mathematics, Faculty of Science and Technology, Surindra Rajabhat University, Thailand

E-mail: satidkku07@gmail.com

²Department of Mathematics, Faculty of Science and Technology, Surindra Rajabhat University,

Thailand

E-mail: promsukan@hotmail.com

Abstract

In this paper, we studied and found the new matrices of 3×3 , which it have similar properties to Fibonacci Q – matrix. Moreover, we studied and found the matrix formula

$$Q^{n}\begin{bmatrix} 0 & 2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{n}\begin{bmatrix} 0 & 2 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} F_{n} & L_{n} \\ F_{n+1} & L_{n+1} \\ F_{n+2} & L_{n+2} \end{bmatrix}$$

when F_n and L_n are Fibonacci and Lucas sequences, respectively.

Keywords: Fibonacci sequences, Lucas sequences, Q-matrix

1. Introduction

The Fibonacci sequences is the sequence of interger F_n defined by the initial values $F_0 = 1$, $F_1 = 1$ and the recurrence relation (Koshy, 2001).

 $F_n = F_{n-1} + F_{n-2}$

for all $n \ge 3$.

The frist few values of F_n are 1,1,2,3,5,8,13,21,34,55,89,144,...

The Lucas sequences is the sequence of interger L_n defined by the initial values $L_0 = 2$, $L_1 = 1$ and the recurrence relation (Koshy, 2001).

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$$L_{n} = L_{n-1} + L_{n-2}$$

for all $n \ge 3$.

The frist few values of L_{h} are 2,1,3,4,7,11,18,29,47,76,123,199,...

The Fibonacci Q – matrix was first used by Brenner (Brenner, 1951), and its basic properties were enumerated by King (King, 1960).

In 1981, Gould showed that the Fibonacci Q – matrix is a square 2×2 matrix of the following form,

1

1 0

0

The following property of the *nth* power of Q – matrix was proved

(Gould, 1981).

In 1985, Honsberger showed that the Fibonacci Q – matrix is a square 2×2 matrix of the following form,

F_2	F_1	IT
F_{1}	F_0	0

The following property of the *nth* power of Q – matrix was proved

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(Honsberger, 1985).

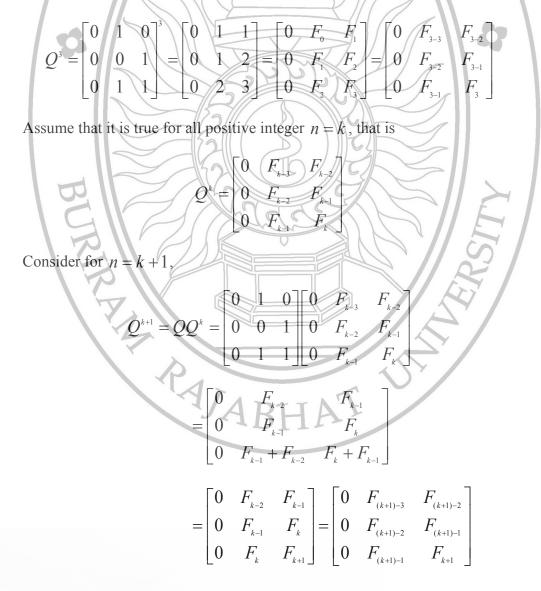
In this paper, we studied and found the new matrices of 3×3 , which it have similar properties to Fibonacci Q – matrix.

2. Main Results

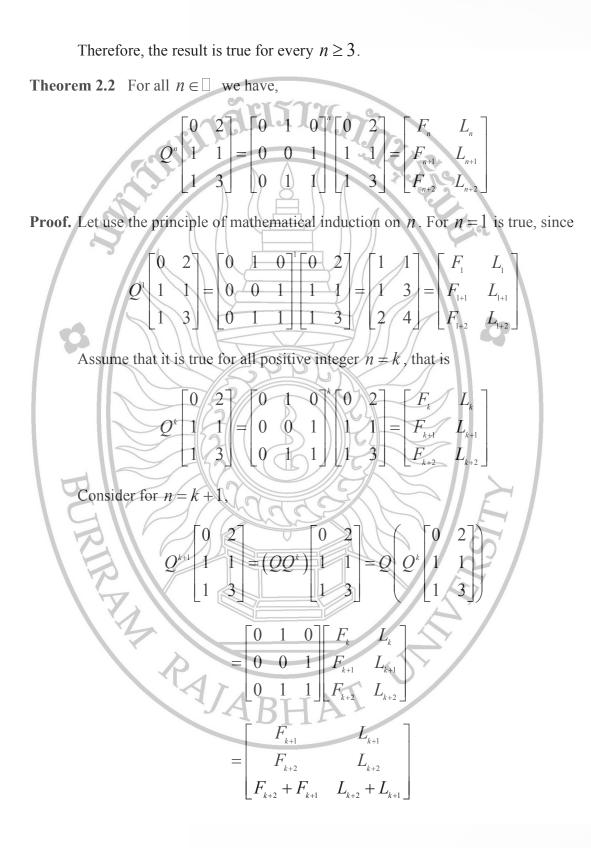
In this study, we studied and found the new matrices of 3×3 , which it have similar properties to Fibonacci Q-matrix. Moreover, we investigate the new property of Fibonacci and Lucas number in relation with the Fibonacci and Lucas matrices formula. We have the following theorem.

Theorem 2.1 If
$$Q = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
 then $Q^n = \begin{bmatrix} 0 & F_{n-3} & F_{n-2} \\ 0 & F_{n-2} & F_{n-1} \\ 0 & F_{n-1} & F_{n} \end{bmatrix}$ for all integers $n \ge 3$

Proof. Let use the principle of mathematical induction on n. For n = 3 is true, since



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$$\begin{aligned} & = \begin{bmatrix} F_{n+1} & L_{n+1} \\ F_{n+2} & L_{n+2} \\ F_{n+1} & L_{n+1} \\ F_{n+1} & L_{n+$$

Completes the proof.

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3. Conclusion

In this paper, we studied and found the new matrices of 3×3 , which it have similar properties to Fibonacci Q-matrix. Moreover, we investigate the new property of Fibonacci and Lucas number in relation with the Fibonacci and Lucas matrices formula.

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